

**EDUC 411  
TEACHING CHILDREN MATHEMATICS<sup>1</sup>  
Fall 2019**

<b>Instructors</b>	Deborah Loewenberg Ball	Carolyn Hetrick
<b>Campus office</b>	4002 SEB	2211 SEB
<b>Email</b>	<a href="mailto:dball@umich.edu">dball@umich.edu</a>	<a href="mailto:hetrickc@umich.edu">hetrickc@umich.edu</a>
<b>Phone</b>	734.972.4793	510.316.7842

**Class meetings:**

Room 4212  
School of Education

Tuesdays  
9:00 – 11:50 a.m.

Exceptions:  
No class on: October 15 (Fall Break)

**Office hours:** By appointment

We will use both Canvas announcements and email to communicate with you; we encourage you to do the same with us. Please also feel free to use text messages to Deborah: 734-972-4793. We will use Canvas (course site: [EDUC 411 Fall 2019](#)) to post documents and announcements from class. You will use Edthena to submit most assignments (group name: ED411 Teaching Children Mathematics Fall 2019).

**COURSE POLICIES AND PRINCIPLES**

**Diversity and Respect in the Classroom Community**

In order to create community and spaces where people share their ideas and views and are open to hearing others, and where we seek to challenge and change patterns of marginalization and privilege, the following core principles are fundamental and expected in this class:

- **Respect:** We must respect and value the efforts, identities, capacities, and ideas that each person brings into the space. We call people their chosen names and we make the effort to learn and to say their names as they wish them said. We are fully present in class, which means actively attending when people are speaking and thoughtfully considering and responding to others' ideas and contributions. This includes adding on to and building on others' ideas and raising challenges or questions.
- **Curiosity and openness:** We must all be open to alternative views, experiences, and perspectives, and curious to learn about and from one another. Freedom to express ourselves—a fundamental civil and human right—excludes expressions that commit or encourage violence or trauma toward others. I do not invite racist, sexist, classist, and, generally bigoted ideas, nor am I inviting tolerance or respect for such ideas. Judgments about this are part of the responsibility that a free and just society entails.
- **Diversity:** We stand for the goals of diversity, inclusion, justice, and equity expressed in our school community's statement of institutional commitments: <http://www.soe.umich.edu/diversity/>. Acting on these commitments in our day-to-day work together means that we each must cultivate awareness of our own biases

<sup>1</sup> The work over two decades by the Mathematics Methods Planning Group (MMPG) at the University of Michigan School of Education Elementary Teacher Education program has influenced and contributed to the development of the Fall 2019 syllabus. We acknowledge in particular the contributions over time of Meghan Shaughnessy, Tim Boerst, Laurie Sleep, Merrie Blunk, Hyman Bass, Rosalie DeFino, Susanna Farmer, Nicole Garcia, Kristi Hanby, Lindsey Mann, Darius Robinson, Sabrina Salazar, Sarah Kate Selling, Charles Wilkes, and Amber Willis.



and perspectives. Actively advancing diversity, equity, inclusion, and justice requires that we are mindful of our ways of being, listening, talking. Being cognizant of our own biases and perspectives and actively working to advance diversity, equity, inclusion, and justice will require each of us to critically interrogate the materials, ideas, structures, and contexts we examine, and the ways in which we examine them in our work together.

- **Collective work to develop critical consciousness and action:** As educators, we seek to see and confront biases and oppressive patterns more clearly, and to bring them up to one another for consideration in our class. Because we are all steeped in a society pervaded by oppression and bias rooted in race, class, gender, and ableism, we are all on a trajectory of learning about these issues and will never “arrive.” In our class, developing our capacity to counter these patterns is a collective undertaking. Raising questions, noticing, calling attention to patterns of marginalization, whiteness, and class-based bias is something we do together to become more capable of being teachers who can act to disrupt these pervasive patterns in practice and structural and systemic patterns in schools.

### Professional Behavior

As senior interns preparing for the professional work of teaching, this course places a value on your professional behavior and work:

- You should be prepared for class (readings, tasks, writing) and be ready to begin class promptly at 9:00 a.m.
- You are expected to turn in all work for this course on time, just as you will have to do as a teacher. If extenuating circumstances prevent you from turning in an assignment on time, please contact us prior to the deadline to make professional accommodations.
- Your work should be carefully done, proofread, and presented in a professional manner.
- Your contributions to our discussions and work in class are crucial to the quality of the course, and you are expected to consider how you are contributing and interacting with colleagues and instructors.

### Appreciation

Our opportunities to learn together in this course owe a great deal to the work of many others who labor to ensure that classes are held in clean and heated rooms, that the technology works to support our learning, and that we have the supplies we need, and access to the materials and resources we need.

We especially would like to thank Tina Sanford in the Educational Studies office, Meri Tenney Muirhead and Mariella Ortiz-Reyes in the Elementary Teacher Education program office, Mike Napolitan and Daniel Adkins in the SOE Facilities office, and Joanna Elliot and Kaisa Ryding in Instructional Technology Services. These individuals and many unnamed others across campus are often invisible to us and are disproportionately people of color and low-income people, while their labor creates comfort and security for our school. In fact, their work is successful when it simply happens without attracting attention, yet they are often not accorded the respect and gratitude they deserve. Please join me whenever you can in expressing thankfulness for their skill and resourcefulness in making it possible to teach and learn here.

### Accommodations for Students with Disabilities

If you need an accommodation for a disability, please let me know as soon as possible. Some aspects of this course—the assignments, the in-class activities, and the way the course is taught—may be modified to facilitate your participation and progress. As soon as you make me aware of your needs, I will work with the Office of Services for Students with Disabilities (SSD) to help me make appropriate academic accommodations. SSD typically recommends accommodation through a Verified Individualized Services and Accommodations (VISA) form. Any information you provide is private and confidential and will be treated as such. SSD contact information: 734-763-3000; <https://ssd.umich.edu/>

### COURSE FOCUS, PREMISES, AND LEARNING GOALS

Teaching Children Mathematics is a nine-week course focused on teaching mathematics to elementary and middle school students. The central goal is to learn to teach mathematics in ways that are responsible to the goals of the curriculum, responsive to children’s resources and identities, and imaginative and resourceful about the domain we call “mathematics.” You can learn to teach this subject well as a first-year teacher and still be set up to learn on an ongoing basis from your own practice.

Mathematics is a subject area in which students can become deeply engaged and actively fascinated. However, math in school has too often been a domain in which some children are seen as “smart” and others as “behind” or “not good at math.” This has not been random, and most often affects Black and brown children, girls, children who speak



languages other than English at home, and children who live in low-income environments. Feeling “not smart” in math can have significant consequences for children, both for how their teachers and peers see them and for how they see themselves.

Because schools in the United States have historically been inequitable, learning to teach mathematics as a professional demands learning to use one’s discretion in ways that disrupt rather than reinforce patterns of marginalization. It is important to acknowledge that, based on their intersectional identities, people have had different experiences in and out of schools, and thus, as teacher candidates, you too bring different resources that can be built on or that need to be challenged or even unlearned, in order to learn to teach in an inequitable society. This course will help you see and understand the “normalized” patterns that shape these outcomes for children, and develop things you can do, even as a beginning teacher, to not replicate these patterns in your own classroom.

The premises below are fundamental to the design of this course and our work together. We believe that it is important to make them explicit:

- Racism has prevailed throughout our nation’s history and persists.
- Teaching either maintains or confronts patterns that preserve racism, other systems of oppression, and their intersections.
- Children have rights to be respected and regarded as humans (e.g., to speak, listen, and be heard; to write only what makes sense, to have time to think and reflect)
- Dominant conceptions of mathematical competence in school are narrow, reductionist, and exclusionary.
- Mathematics is a human endeavor and for children to learn it, they must engage in making mathematics.
- Discussions in math class can offer significant opportunity to interrupt dominant patterns.
- Discussions in math class can make it possible for children to experience learning as a collective undertaking.

Based on all of what we have written above, what will we do together to support your development as a teacher?

We have designed the course to support your learning. Our work will involve *decomposing* what is involved in each of the high-leverage practices (HLP) that are in focus this term, so that you can see the specific elements of doing them well. We will show you examples of the HLP, *representing* it through the use of *video* and through *modeling* it in class. We will also *approximate* carrying out parts of these practices, using *rehearsals* which are structured ways of trying out particular moves or approaches and getting close feedback. Rehearsing like this can make your opportunities to practice in real classrooms much more valuable, because you will already have some foundational skill and nuanced judgment before you try to pull this off in real time.

We will also work on knowing mathematics in the specialized ways needed in teaching. Teaching mathematics requires a great deal of specialized mathematical knowledge—knowledge that is different from what it takes to do well in a math course as a student yourself or to be good at other jobs that require mathematics. Your own understanding, fluency, and comfort with mathematics will be important to your effectiveness as a teacher. In preparing to teach, you will be shaping the mathematical goals of activities, anticipating the varied ways students might respond, and preparing mathematically for what might happen as a lesson unfolds. You will prepare good questions to ask. You will generate easier as well as harder versions of problems, either as back-up plans or as ways to focus or extend students’ work. You will need a keen sense of the complexity of particular mathematical ideas, and ways they can be scaffolded for students’ learning. When your students have trouble, you will need to be able to figure out what they are doing mathematically, and whether it makes sense.

This semester we will work in particular on place value and operations with whole numbers and decimals, which are central topics in the K-8 curriculum and the way they are taught can either help students succeed or hold them up. We will also deal with concepts in number theory and some work on classification and logic. We will unpack mathematical practices that matter for using and thinking with math. These include things such as making sense of what problems are asking, persevering with challenging work, building and critiquing mathematical arguments, using language with care, and noticing and using mathematical structure. We know you have had courses in the math department; our work will focus on the kinds of understanding and ability to interpret and communicate that are crucial to teaching well.

Both the mathematical understanding and your commitment to challenge and change the racialized and gendered patterns that permeate math classrooms depend on your actual skill with practices of teaching, as well as conscious and deliberate enactment of the foundational ethical obligations of the profession you are entering:



1. To care for and demonstrate commitment to every child
2. To develop and continually work to improve one's practice
3. To work deliberately to ensure equitable access to learning
4. To learn about and demonstrate awareness of, sensitivity to, and appreciation for diversity and to draw on it as a resource in teaching
5. To enact through concrete actions the fundamental belief in the capacity of every person to learn
6. To take responsibility and work assiduously for equity
7. To exercise carefully the power of one's role as a teacher
8. To treat children, families, and community members with respect and generosity
9. To represent the ideas of academic disciplines with integrity

During this course, you should further develop your capability to carry out these practices in the context of teaching mathematics, as detailed below.

Course Focus	Corresponding High-Leverage Practices
(1) Representing and explaining mathematical ideas (emphasis on number concepts and operations)	<ul style="list-style-type: none"> <li>• Explain mathematical content</li> <li>• Choose and use representations, examples, and models of content</li> <li>• Use student representations and explanations to include students often not represented in curriculum or otherwise marginalized in mathematics classrooms</li> <li>• Narrate key aspects of representations and explanations in ways that make important mathematical ideas apparent and connect student thinking to conventional representations and explanations</li> </ul>
(2) Leading whole class discussions about mathematics	<ul style="list-style-type: none"> <li>• Lead a whole class discussion</li> <li>• Pose questions</li> <li>• Listen to and interpret students' ideas</li> <li>• Establish norms and routines for classroom discourse and work that are central to the content</li> <li>• Support and help students appreciate the collective nature of mathematical work</li> <li>• Support students to contribute and learn</li> <li>• Provide feedback to students: Position students as contributing and smart, with attention to patterns that reproduce inequality and oppression of particular groups</li> <li>• Support productive student behavior and engagement by engaging children's strengths and using inclusion-oriented moves, and not common ones that too often exclude and marginalize</li> </ul>
(3) Assessing students' mathematical knowledge and skills	<ul style="list-style-type: none"> <li>• Select and use specific methods to assess students' learning</li> <li>• Enact asset-based attention to students: See what students do know and what they can do, and understand common patterns of thinking about specific mathematical content</li> <li>• Understand and see a full range of mathematical competence in the many different forms it takes</li> </ul>
(4) Planning mathematics instruction	<ul style="list-style-type: none"> <li>• Choose, appraise, and modify tasks for a specific learning goal</li> <li>• Develop and use deliberate equitable practices</li> <li>• Anticipate what could arise that would reproduce inequity and oppression of particular groups and plan ways to avert or disrupt such possibilities</li> </ul>



<p>(5) Attending to policies and practices in the field and communicating with other professionals</p>	<ul style="list-style-type: none"><li>• Cultivate ongoing awareness of policies, popular practices, and resources</li><li>• Use a set of questions to enact a critical and curious mindset to appraise ideas, practices, and policies while considering integrity of the content, patterns that reproduce or disrupt inequity</li><li>• Discuss contemporary issues with other professionals</li></ul>
--	--

- 1) *Representing and explaining mathematical ideas:* Mathematical ideas and processes can be represented in many ways. Having a wide repertoire of mathematical representations and knowing how to establish the equivalence of different representations are important not only in doing mathematics but also in teaching it. For example, with a given task or explanation, one representation of the mathematics may be better suited than another to solve the problem or to make an explanation clear. Considering representation and explanation is also fundamentally about equity and being attuned to how choices about these can marginalize or include particular groups or children, and we will support you to connect these aspects of teaching in practice. By the end of this course, you should be able to use representational resources in several mathematical domains, particularly number concepts (with a focus on whole numbers and decimals) and the operations of arithmetic (addition, subtraction, multiplication, division). You should be able to construct careful correspondences between physical and pictorial representations and some of the key ideas and procedures of the K–8 curriculum. You also should have developed more general skills for representing and explaining mathematical ideas in domains beyond those in this course or those we worked on in Children as Sensemakers last semester.
- 2) *Leading class discussions about mathematics:* You have already worked on developing your skills at leading discussions—in small groups last semester and in other subject areas, especially English language arts. Discussions can play a central role in the learning and teaching of mathematics. Discussions create opportunities to work collectively on mathematical ideas and practices. Students benefit from listening to, interpreting, appraising, and using others’ thinking. Productive mathematics discussions can be held in any classroom setting, across a wide variety of mathematical topics and curriculum materials, and across grade levels. However, productive mathematical discussions do not happen by chance. Leading productive discussions involves posing questions to elicit, engage, and challenge students’ thinking, supporting students’ talk, and strategically introducing and connecting ideas. Discussions can provide crucial opportunities to position children as smart and contributing in ways that do not reproduce normal patterns that tend to marginalize particular groups. By the end of this course, you should be able to conduct a productive mathematics discussion that combines serious attention to the quality of the mathematical work, equity, and careful skill with nuances of particular teaching practices.
- 3) *Assessing students’ mathematical knowledge and skills:* Finding out what students know, how they work on mathematical tasks, and their dispositions toward the subject is essential for responsible and responsive teaching. This involves using a variety of assessment practices to improve your teaching, documenting students’ learning, and communicating about assessment with students, parents, and other educational stakeholders. Assessment encompasses much more than grading and testing. It includes interacting with students as they are learning, pausing to document what students are saying, and noticing patterns in students’ work. Assessment is an opportunity to identify strengths in children’s thinking that can be built upon. Doing this well depends on flexible ways of understanding the mathematics, seeing children as inherently smart and capable (especially students who are members of non-dominant groups who are often positioned as “behind” or “struggling”). By the end of this course, you should be able to use multiple assessment techniques and make use of these to support students’ learning.
- 4) *Planning mathematics teaching:* Good mathematics teaching does not happen by chance. Effectively connecting students with mathematics requires *planning*—deliberate design and preparation. This is true of all subjects, and so we want to connect with more general principles of planning that you have been developing and also learn things that are especially important for teaching math. There are special things to do as you plan math lessons that are particular to the teaching of math. One aspect that can easily be taken for granted is taking time to make sure you feel ready with the content itself. Planning also involves anticipating patterns that are likely to arise, patterns that reproduce patterns of racism, sexism, classism, and ableism, and planning specific alternatives and interventions to offset these patterns. By the end of this course, you should have further developed skills with planning that you have been developing throughout



the program that will allow you to take into account both your students and the mathematics as you prepare for day-to-day instruction.

As in other courses in the elementary teacher education program, our work will be “practice-based” in four senses of the phrase. First, you will be participating in a common practice; our class activities, discussions, and interactions offer us opportunities to study practice from the inside. You can learn by paying attention to and analyzing our interactions together. Second, we will study records of practice to learn the work of teaching. These records—video records of lessons, students’ work, and teacher’s plans, materials, and reflections—make it possible for us to study classroom mathematics, the work of teaching, and students. Third, we will practice together and critique the use of specific teaching techniques to improve enactment and deepen understanding. Fourth, the major assignments and assessments will involve carrying out specific practices in your field placement. Our expectations for your performance are that you demonstrate the skill and knowledge levels appropriate to be developing into a responsible and responsive beginning teacher who can act in ways that recognize and interrupt common patterns that marginalize Black and brown children, girls, children who speak languages other than English at home, and children who live in low-income environments. This includes your practice in teaching children in communities that are predominately white and middle class.

## COURSE MATERIALS

### *Required materials:*

For this course, you will need to have a video recorder and tripod for the field-based assignments. There is no required text for the course.

You will be reading articles and other materials (e.g., curriculum materials) which will be posted on Canvas.

## ASSIGNMENTS AND EVALUATION

You will have three types of assignments in this course: weekly assignments, professional practice assignments, and culminating performance assessments.

1. **Weekly assignments:** There will often be small assignments to prepare for our work in class. These will be related to the understanding and practices of teaching that you are developing this term and will sometimes include particular reading assignments. These assignments will also be designed to help you prepare for upcoming work in class. They will not be graded, but they are designed to support your development as a teacher and completing them as scheduled is a requirement of the course. You will share work you do on these in class and get feedback from others.
2. **Professional practice assignments:** These assignments provide opportunities to try out and get feedback on the specific aspects of teaching on which we are working this term. You will carry out specific parts of teaching using what we are learning in class and then turn in a video and analysis of your practice. You will receive feedback aimed at helping you improve your practice. The point is to learn to do teaching. You will analyze your teaching so that you can develop your ability to learn from and improve your practice.

Professional Practice Assignment	Brief Description	Date of Field Work
Discussion #1	To take stock of your own skills in leading math discussions, you will lead a 15-minute discussion about a short mathematics problem. I will provide a mathematical task for you to use for this discussion. We will meet in grade-level groups to work on this in class, but you will need to plan in more detail how you will go about the short discussion.  You will analyze your discussion using the decomposition of discussion-leading to identify your current strengths and areas that need more learning, to frame your work for this course.	In field placement October 9–17
Discussion #2	To develop your skills and moves for leading a whole class discussion, you will lead a 15-minute discussion about a short mathematics problem.	In field placement October 30– November 7



Discussion #3	To develop your skills and moves for leading a whole class discussion, you will lead a 20-minute discussion about a short mathematics problem. You will confer with your mentor teacher about a problem that would be most appropriate for your class.  You will develop and try out an “end-of-class check” (sometimes called an “exit slip”) at the conclusion of the discussion.	In field placement November 13–27
Representing Mathematical Ideas #1	To develop your skills in using manipulatives to model a standard computational algorithm, you will video record yourself modeling an algorithm worked on in the course. This assignment does not involve work with children.	<i>Not done with children</i> November 5
Teaching an Entire Mathematics Lesson	To prepare for your work during the student teaching semester, you will plan and teach an entire mathematics lesson. Before teaching the lesson, you will meet with your field instructor to discuss your plan.  Your enactment of the lesson will not be graded because it will be your first chance to teach a full mathematics lesson as a part of this course. It will be considered one of your weekly assignments.	In field placement November 6– December 5

3. Culminating performance assessments: To assess your progress and ability to perform these practices, you will also complete *culminating performance assessments* focused on the central teaching practices in this course.

Culminating Performance Assessment	Strand of Course	Short Description of Assignment	Setting
Discussion #4	Leading a whole class discussion about mathematics	Lead a whole class discussion using curriculum materials from your MT.	In field placement December 2–6
	Assessing students’ mathematical knowledge and skills	Design and implement a short prompt to use with students as a quick written assessment at the end of a discussion (or lesson), and then analyze what students produce.	
Representing mathematical ideas #2	Representing mathematical ideas	Use manipulatives to model one of the standard computational algorithms worked on in the course.	<i>Not done with children</i> December 3

### Grading

This is a professional course. The standards of performance are tied to your knowledge and enactment of practice *and* to standards you will be expected to meet as a beginning teacher: meticulous preparation, appropriate use of professional knowledge, careful consideration of alternatives, genuine curiosity about ideas and about learners, exercise of professional judgment, collegial work on teaching, respect for children, peers, and other professionals, analysis and reflectiveness, skills of ongoing professional learning, clear expression, organization, and timeliness.

Just a few months from now you will no longer be an undergraduate student, but a beginning teacher. Your actual skills, understanding, and ability to be responsible and responsive, to be someone who can see patterns that reproduce inequities and to actively counter these in your practice, to teach mathematics in ways that make sense to and engage your students will matter. Given this, your grades are structured to emphasize the aspects of your work in this course that will matter most in your beginning teaching (namely, your enactment of math teaching).

Your final grade will be determined as follows, based on the quality of your enactment of each of the course requirements. We will provide detailed guidance about what is involved in skillful performance of each of these, including feedback on your performances. We expect that you will take up this feedback and demonstrate that you are actively working to develop your teaching skill over time.



<i>Assignments</i>	<i>Percentage of Final Grade</i>
Discussion #2	10%
Discussion #3	15%
Representing mathematical ideas #1	15%
Representing mathematical ideas #2 (Culminating Performance Assessment)	15%
Discussion #4 (Culminating Performance Assessment)	
• Leading a discussion	15%
• Assessing students' mathematical knowledge and skills	15%
• Self-appraisal of teaching progress	15%

### Academic and Professional Integrity

We expect that you will submit original work and will appropriately cite others' work referenced in assignment submissions. If you are unsure about how to correctly cite others, please ask. Please refer to the following website for U-M policies and procedures regarding academic and professional integrity:

[http://www.soe.umich.edu/file/academic\\_integrity/](http://www.soe.umich.edu/file/academic_integrity/)

### Tentative Course Schedule with Major Assignments

*Readings and week-to-week assignments will be provided throughout the term and will be discussed in class and also posted on Canvas.*

<b>Week</b>	<b>Class focus</b>	<b>Major assignments</b>
CLASS 1 <b>October 8</b>	Orientation to the course; Preparing to lead Discussion #1	Discussion #1 assigned
CLASS 2 <b>October 22</b>	Working on place value concepts with children; Disrupting problematic patterns with handling unexpected student responses and teaching rules without meaning	<b>Discussion #1 due</b>
CLASS 3 <b>October 29</b>	Addition (explaining core content and choosing representations, contexts, and examples); Orienting students to the thinking of others	Discussion #2 assigned Representing Mathematical Ideas #1 assigned
CLASS 4 <b>November 5</b>	Subtraction (explaining core content and choosing representations, contexts, and examples); Selecting tasks for discussions; Setting up a task	<b>Representing Mathematical Ideas #1 due</b>  Teaching an Entire Mathematics Lesson assigned (discuss with your Mentor Teacher)
CLASS 5 <b>November 12</b>	Multiplication (explaining core content; choosing numerical examples); Launch work on assessment	<b>Discussion #2 due</b> Discussion #3 assigned
CLASS 6 <b>November 19</b>	Multi-digit multiplication (explaining core content and choosing representations, contexts, and examples); Analyzing assessment information	
CLASS 7 <b>November 26</b>	Division and divisibility (explaining core content); Concluding discussions	Discussion #4 assigned
CLASS 8 <b>December 3</b>	Multi-digit division (explaining core content and choosing representations, contexts, and examples); Communicating with caregivers about students' mathematics learning and progress	<b>Discussion #3 due</b> <b>Representing Mathematical Ideas #2 due</b>
CLASS 9 <b>December 10</b>	Extending place value concepts to decimals and large numbers (explaining core content and choosing representations, contexts, and examples)	<b>Discussion #4 due</b> Teaching an Entire Mathematics Lesson Due

