

# Could Instructional Materials Improve Mathematics Teaching and Learning?

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Learning Session with the Bill & Melinda Gates Foundation

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# Before we go there . . .



# Three premises frame our work today

1. Mathematics education improvement is not a new project. Let's look back—and use—what we have learned from past efforts.
2. Instructional materials are essential for good teaching, but they cannot be a panacea. They can be a partner in improving mathematics learning.
3. Enactment is different from “implementation” and demands particular support.

repairing longstanding  
patterns of racial inequity

building the STEM  
workforce

# What are we after?

improving mathematics  
achievement by U.S. students

growing a generation  
of adults who don't  
disidentify with math

broadening our conception  
of mathematics and  
mathematical competence



# But we have been here before.



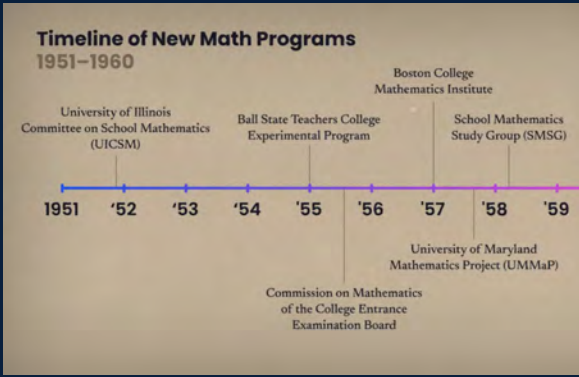
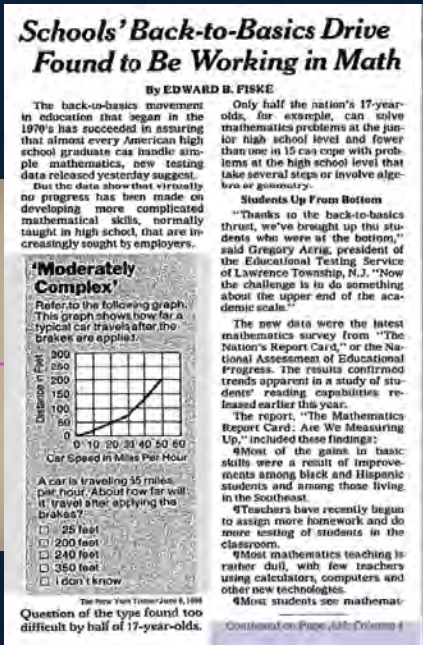
# But we have been here before.



many times



A 1953 issue of *The Mathematics Teacher* stated that “competence in mathematics widespread among our people is essential for the preservation of our society”.



1950-60s

1970-1985



1990s

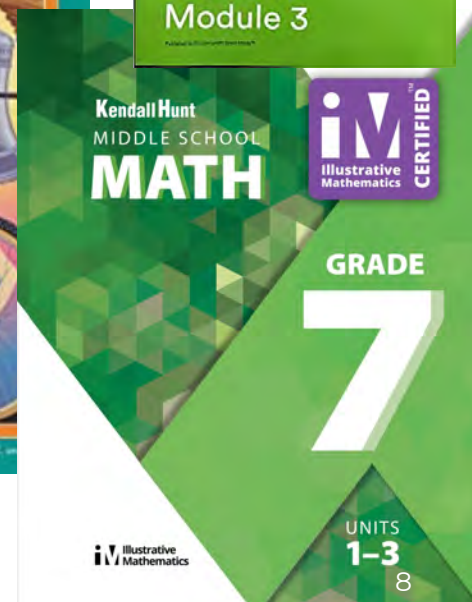
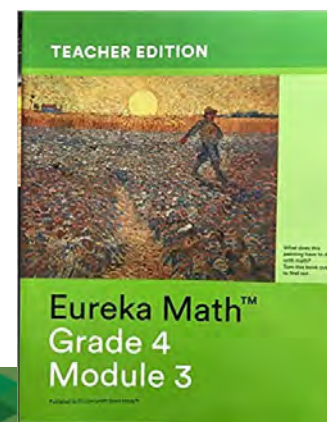
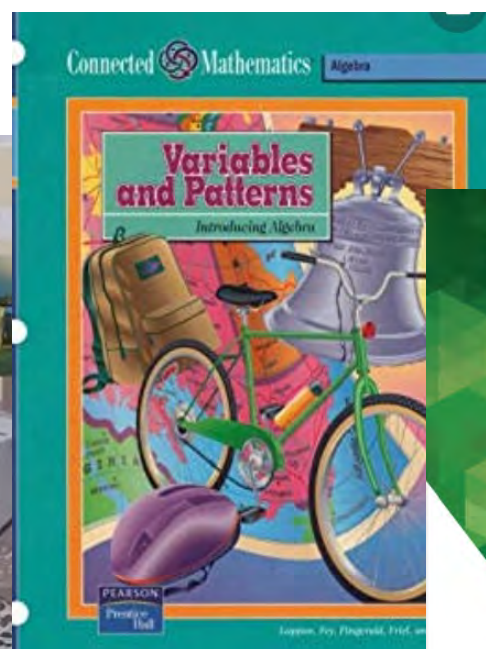
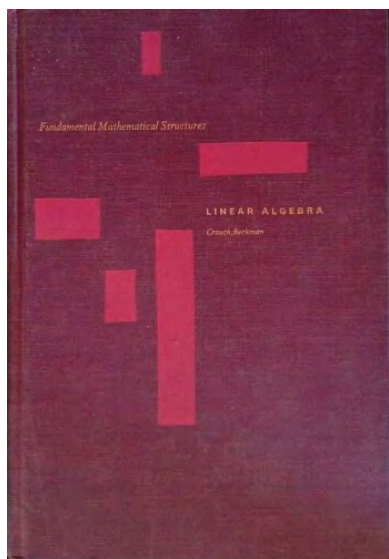


2010-



# What have been the patterns?

repeated investments in designing and implementing new curriculum materials



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# What else have been the patterns?

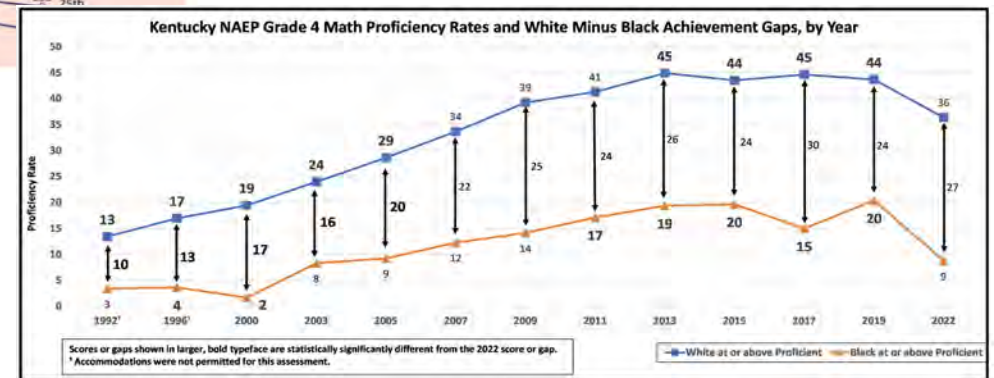
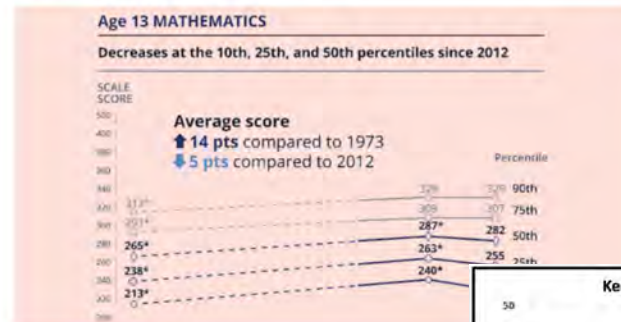
recurrent worries about achievement data

## U.S. STUDY SHOWS PUPIL ACHIEVEMENT AT LEVEL OF 1970

American elementary and secondary school pupils have made some educational progress in recent years, but they are only now reaching the achievement levels of students in 1970, according to a major Federal report issued today, which added that students are unprepared for the complex and demanding world of the 21st century. "Today's children seem to know about as much math and about as much science and read about as well as their parents did at that age about 20 years ago," Education Secretary Lamar Alexander said of the report, "Trends in Academic Progress."

October 1, 1991

## Long-Term NAEP Scores for 13-Year-Olds Drop for First Time Since Testing Began in 1970s — 'A Matter for National Concern,' Experts Say



# And another pattern . . .

lack of understanding of the work of teaching  
and what it takes to develop it



and instead working *around* teachers to improve learning





# What could we do so that we are NOT having this same conversation in 2033?





# What could we do so that we are NOT having this same conversation in 2033?

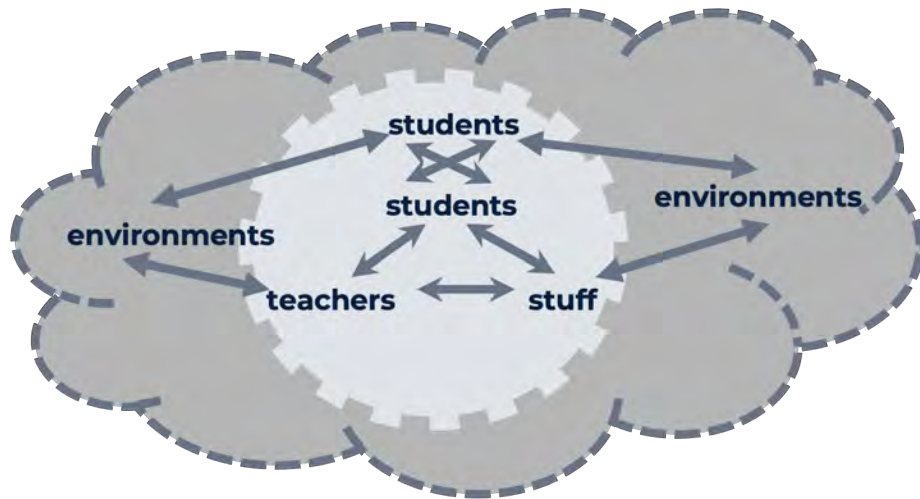
- Let's make a new bet on instructional materials, but with a wisdom rooted in history.
- This would require a deliberate and nuanced focus on the work of teaching mathematics and on what it takes for instructional material to be “high quality.”





But . . . their promise has often remained unrealized.

There is a gap between instructional materials as resources and their use in and for high-quality instruction.



“Curriculum materials could contribute to professional practice if they were designed with closer attention to processes of curriculum enactment.” (p. 7, Ball & Cohen, 1996)

*Further:*

Curriculum materials could play a crucial role in the quality of beginning teaching if we considered the work of use and designed teacher preparation and induction support around learning to use instructional materials.



**This session's question:**

Could  
instructional  
materials  
improve  
mathematics  
teaching and  
learning?

## Proposition:

Instructional materials could improve mathematics teaching, and thus learning, if they were designed, used, and supported as **in partnership** with the real work of teaching.



# From “implementation” to “enactment”

- Policymakers, curriculum designers, reformers all design to make change in practice.
- The “theory of action” is what the designer assumes, implicitly or explicitly, about how things work and interact.
- “Enactment” is what happens as policy is put into practice. Policies are actually made by “street-level bureaucrats”\* as they enact designs in context.

\*Michael Lipsky (1980). *Street-Level Bureaucracy*

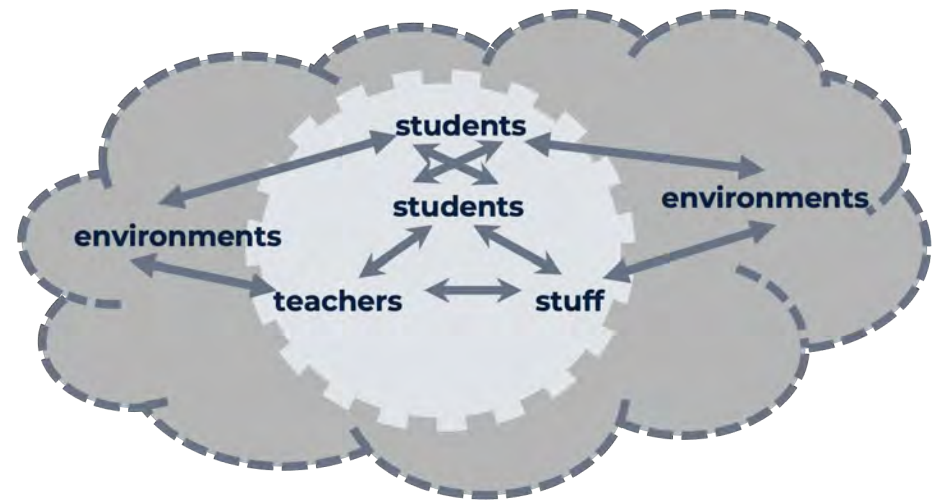
# To figure out what this would mean, we need to consider three questions:

1. In what ways could instructional materials be a key element in strengthening teaching—and what are real pitfalls?
2. What is the work of using instructional materials wisely in teaching?
3. How could teachers be better supported for using and learning from curriculum materials?

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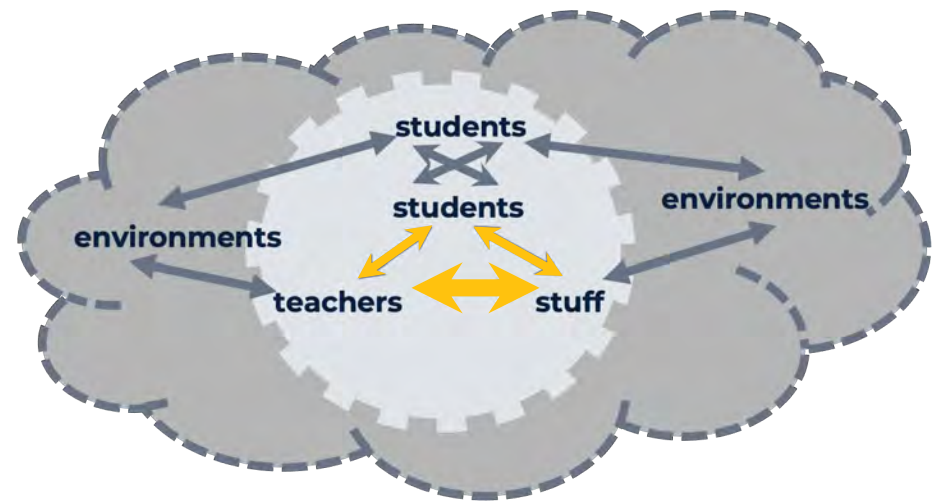


# What can curriculum materials provide for teaching?

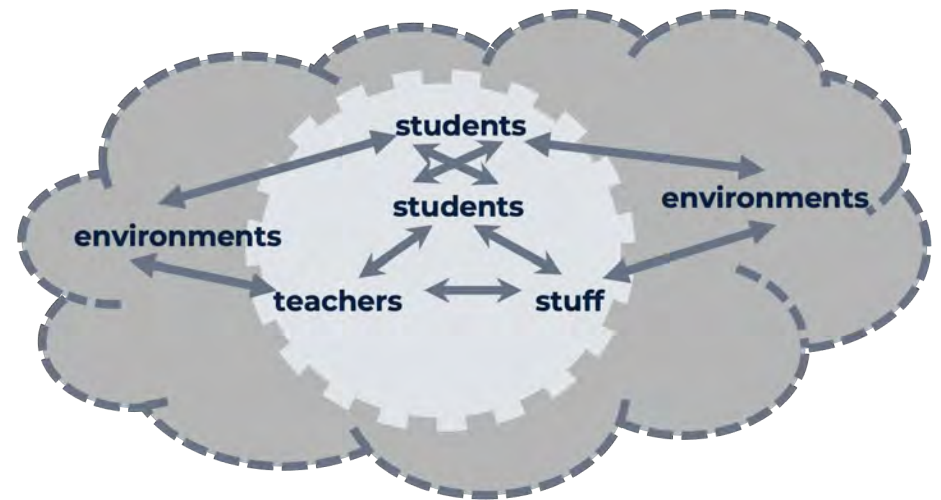


# What can curriculum materials provide for teaching?

- Clear articulation of learning goals aligned to standards
- Explanations of the content from the perspective of content knowledge for teaching (what the core mathematical point is)
- Insights into how students might think and how students' knowledge and experience might interact with the content
- A range of instructional activities and tasks well-coordinated to the learning goals
- Sequenced, learning trajectories, coherence



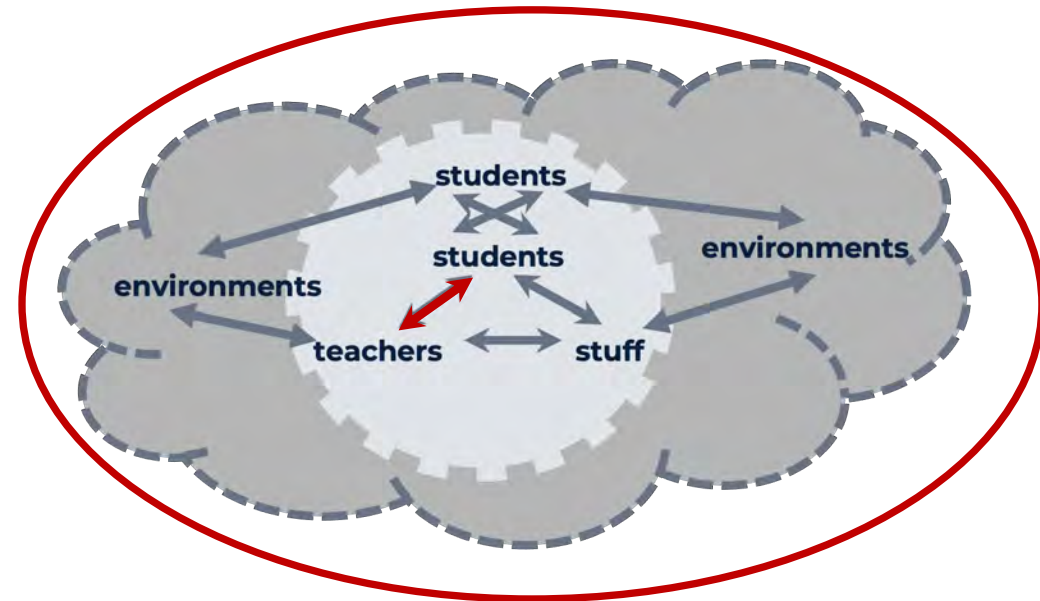
# What can curriculum materials not provide?





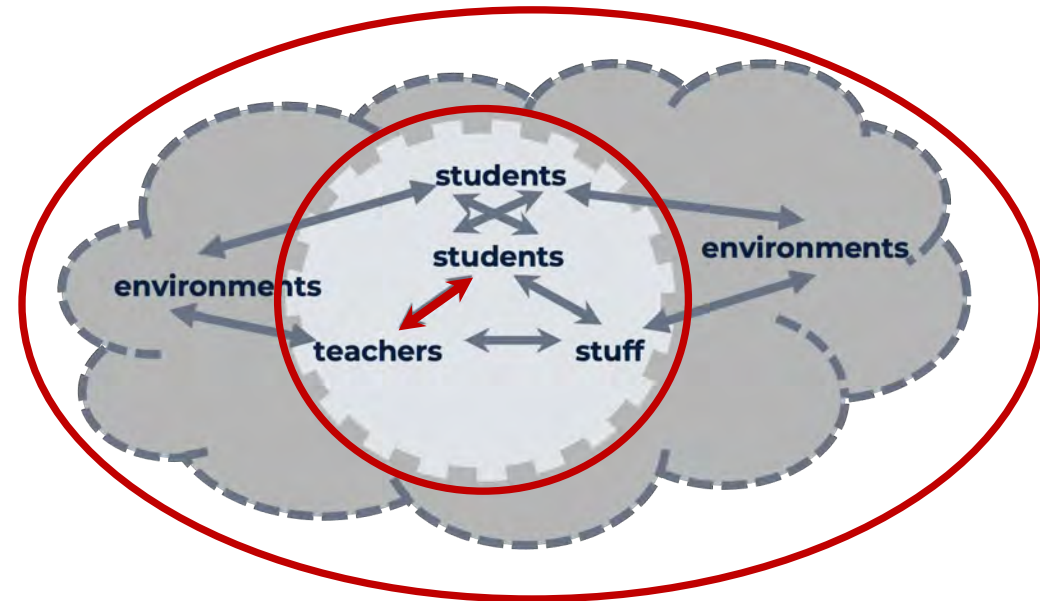
# What can curriculum materials not provide?

- Knowledge about specific students, their contexts, communities, resources, past learning



# What can curriculum materials not provide?

- Knowledge about specific students, their contexts, communities, resources, past learning
- They cannot actually teach —
  - Particular students in one’s own class, the specific classroom culture and norms, and specific moves in the moment to address content, equity, engagement, behavior, etc.



# A brief visit to a math lesson

What part of the rectangle below is shaded gray?



What part of the rectangle below is shaded gray?



# The learning goals

- To identify how to name the unit fraction in an area model, and the central notion of “equal parts”
- To learn how to use and interpret the area model representation for fractions
- To construct, listen to, and critique mathematical arguments
- To develop strong mathematical identities



# Going to the board to show one's ideas



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## COMMON PURPOSES

- To show that there are different ways to solve math problems and sometimes also different solutions
- To provide opportunities for children to practice talking and explaining their ideas

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## CRITICAL PURPOSES

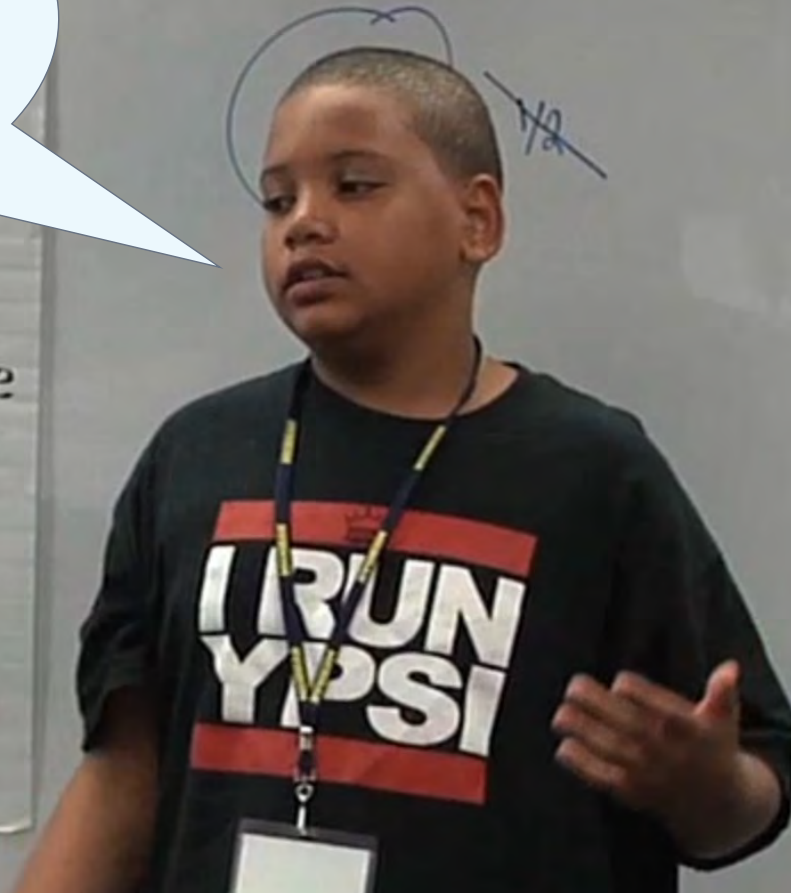
- To represent mathematics as collective work by building mathematical ideas and solutions together
- To disrupt patterns of who and what is seen as competent



VIDEO: Antar explains his thinking

I think it's not a fraction because all of the parts are not equally the same.

What fraction of the rectangle below is shaded gray?



# What is likely to happen next?



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## LIKELY NEXT MOVES

- “Who can help Antar out?”

## RESULT

- Antar, a Black boy, is positioned as not knowing and needing help.



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- “Thumbs up if you agree with Antar; thumbs down if you disagree.”

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- Antar, a Black boy, might face many people disagreeing with him that it is not a fraction.

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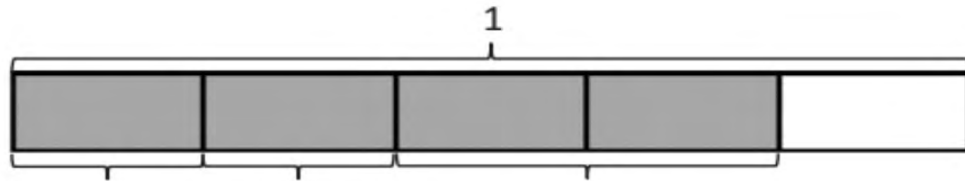
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## RESULT

- Antar, a Black boy, is positioned as not knowing and needing help.
- Antar’s contribution is taken over by the teacher.
- Antar, a Black boy, might face many people disagreeing with him that it is not a fraction.
- Any of these might reinforce narrow and exclusionary views of mathematics.

# What about the mathematical point?

- The big idea here in representing fractions with area models, the parts must have equal area.
- This is often left implicit, with significant mis-learning consequences:



- Antar's statement explicitly opens the key mathematical point.





VIDEO: Gabriella repeats what Antar said

Gabriella: Oh. He said that he doesn't think it's a fraction because not all the parts are equal.



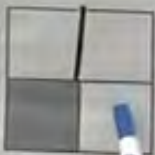
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What fraction of the rectangle below is shaded gray?



What fraction of the rectangle below is shaded gray?



VIDEO: Gabi explains her thinking

$$\frac{1}{3}$$

~~$\frac{1}{2}$~~

Gabi: I divided it down the middle because, since it's not equal, you have to make it equal.



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# The work of teaching has multiple goals and possible effects

## RISKS

- Gabi, a Black girl, is seen as having the “right” answer and Antar, a Black boy, is seen as “wrong”
- These narratives affect Antar and Gabi, but also their classmates
- Math is seen as about getting the right answer

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- These narratives affect Antar and Gabi, but also their classmates
- Math is seen as about getting the right answer

## PRINCIPLES FOR AVERTING THESE RISKS

- Position each student’s contributions at the board as part of a trajectory to construct collective knowledge, language, and ways of justifying, not as competing one-by-one for the “right” answer
- Represent mathematics as collective construction of sense-making and knowledge



Teacher:	Here's a marker. Can you explain your thinking?
Antar:	I think it's not a fraction because all of the parts are not the same shape.
Teacher:	Can you say that one more time to the class?
Antar:	I think it it's not a fraction because all of the parts are not equally the same.
Teacher:	Can someone repeat what Antar said? Very nice. Antar. What did he say? Gabriella?
Gabriella:	Oh. He said that he doesn't think it's a fraction because not all the parts are equal.
Teacher:	Is that what you said?
Antar nods.	
Teacher:	Okay, would someone like to comment on that? Agree or disagree with him?
	<i>Pause.</i>
Teacher:	Okay, let's see, how about Gabi?
Gabi:	I disagree.
Teacher:	What do you think?
Gabi:	I think the fraction is one-fourth.
Teacher:	One-fourth?
Gabi nods.	
Teacher:	Do you want to come up and say why you think it is one-fourth?
	<i>Gabi stands up and walks toward the board.</i>
Teacher:	Antar, do you want to stay there or do you want to sit down?
Antar:	Sit down.
Teacher:	Thank you very much. You did a good job of explaining your thinking. So let's hear what Gabi's thinking.
Gabi:	I think it's one-fourth because, like he said, all the fractions aren't the same, but you can make them the same by dividing them down the middle.
Teacher:	Can you go ahead – want to show us – Here. Here's something you can use for that. Wait one second.
	Instead of drawing it, why don't you just use this. That way, if someone wants to take it off again, they can.
	<i>Gabi puts removable black line down the middle of the figure.</i>
Teacher:	Okay, so now explain what you have done. Talk to the class, okay?
Gabi:	I divided it down the middle because, since it's not equal, you have to make it equal.
Teacher:	And so then you decided?
Gabi:	It's one-fourth.
Teacher:	Okay, so can someone repeat what Gabi said? What she did and what she said? This actually goes very nicely with what you said, Antar, because Antar noticed that the parts weren't equal and what Gabi is doing has to do with equal parts.

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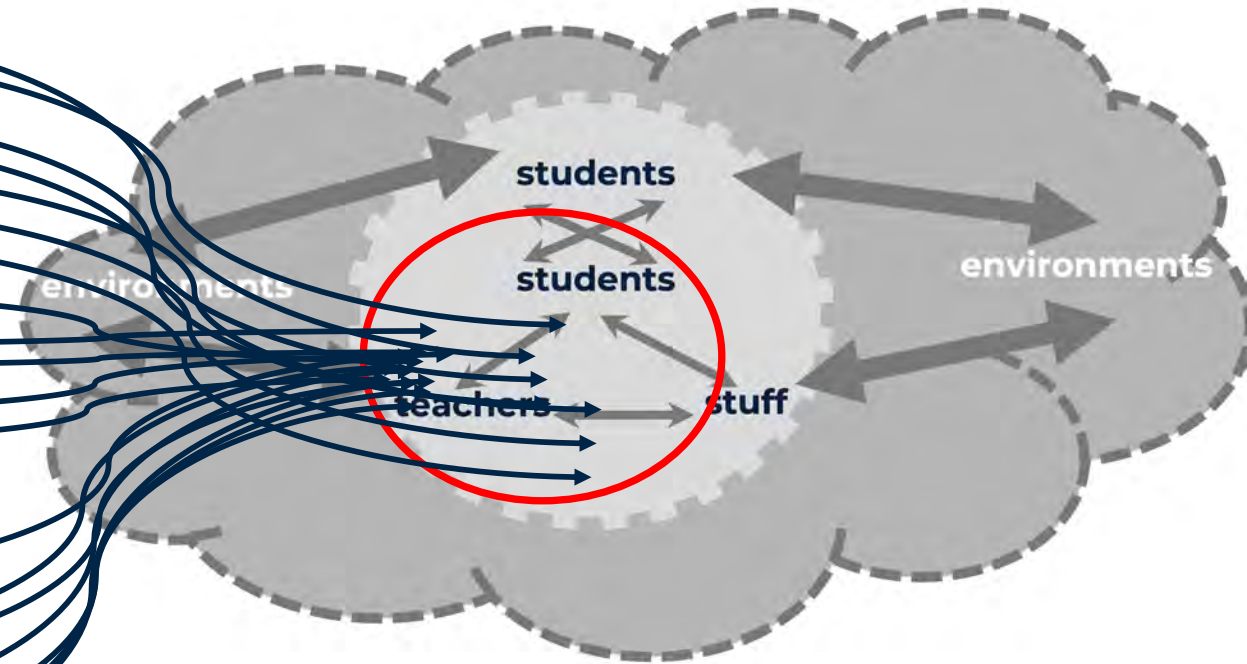
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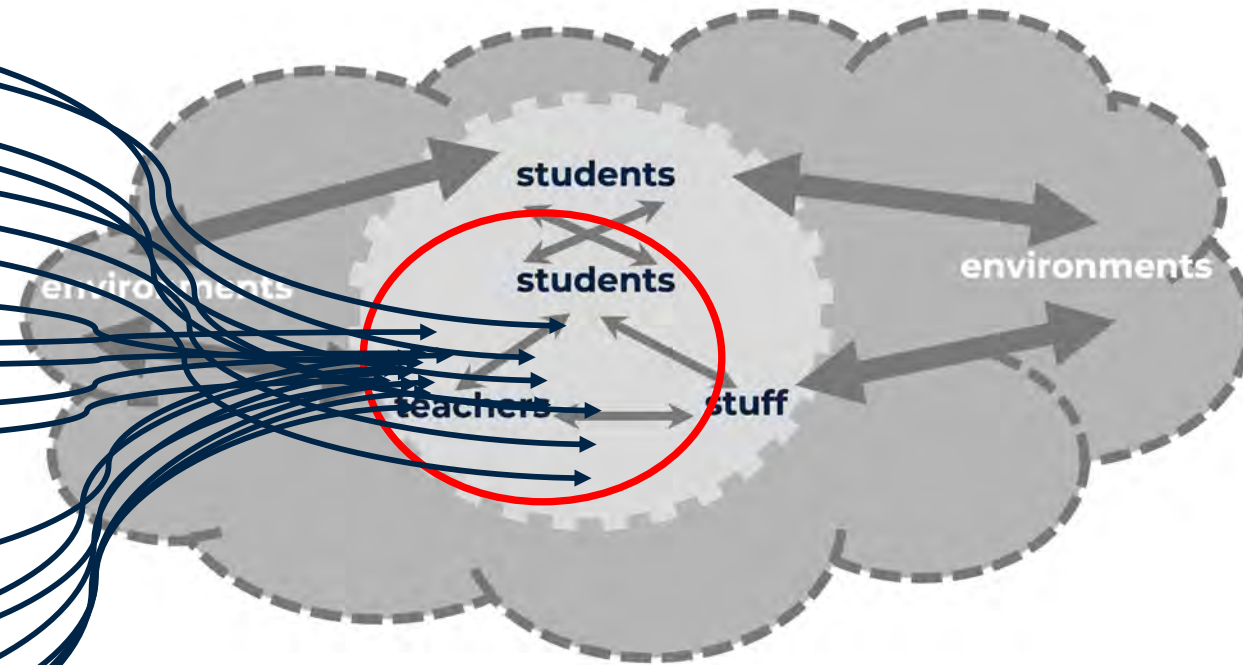


59 in 2:21



# Teaching is dense with 'discretionary spaces'

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59 in 2:21





## 2. What is the work of using instructional materials wisely in teaching?



Let's start with a brief stop in a beginning teacher's classroom.

Serena is in her second year of teaching.

Her school provides a strong standards-aligned textbook for math.

We'll see her use it to teach a lesson on “mean,” or “average.”

# A common learning goal: Averages

CCSS.MATHCONTENT.6.SBP.4 Summarize and describe distributions.


Giving quantitative measures of center (median and/or mean).

(What *is* an “average”? How would you explain it, without just stating the *process* to calculate an average?)



# Using instructional materials to teach this standard

## 10•7 Calculating the Mean

 Objectives To guide children as they calculate the mean of a set of data; and to review the median of a set of data.

### 1 Teaching the Lesson

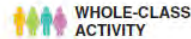
#### Key Concepts and Skills

- Add 2- and 3-digit numbers with a calculator.  
[Operations and Computation Goal 2]
- Collect and organize data.  
[Data and Chance Goal 1]
- Find the median and mean of a data set.  
[Data and Chance Goal 2]
- Use graphs to answer questions and draw conclusions.  
[Data and Chance Goal 2]

#### Key Activities

Children find the median and mean arm spans and heights of adults and children. They practice finding the mean.

### Finding the Median Arm Span of the Class

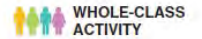


WHOLE-CLASS ACTIVITY

(Math Journal 2, p. 256)

Ask children to describe how they might go about finding the **median**, or typical, arm span from the data on the stick-on notes. **Line up the stick-on notes on the board from the least to the greatest length. Count to the middle.** Have a few children line up the stick-on notes in order on the board and count to the middle. If the number of notes is odd, there will be one note in the middle. If the number of notes is even, there will be two in the middle. In that case, if the two are the same, use that value; if they are not, use the value halfway between. Children record the median arm span for the class in Problem 1 on journal page 256.

### Finding the Mean Arm Span of the Class



WHOLE-CLASS ACTIVITY

(Math Journal 2, p. 256; Student Reference Book, pp. 80, and 83–85)

Explain that children are now going to calculate another typical arm span, the **mean** arm span. Remind them that they found the mean family size in Lesson 10-6 and that **average** is another word for *mean*.

Divide the class into groups of four or five and guide children through the three steps listed below. Each child should do the computations with a calculator, compare his or her results to those of other members of the group after each step, and resolve discrepancies before going on to the next step.

1. Add all the arm-span lengths to find the class total (sum).
2. Count the stick-on notes (number of measurements).
3. Divide the total (sum) by the number of stick-on notes (number of measurements).

The result is the *mean arm span* of children in the class. If the number has decimal places, tell children they can ignore any digits after tenths. Children record the average arm span in Problem 2 on journal page 256.



## Guidance for the instructional activity

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2. Count the stick-on notes (number of measurements).
3. Divide the total (sum) by the number of stick-on notes (number of measurements).

The result is the *mean arm span* of children in the class. If the number has decimal places, tell children they can ignore any digits after tenths. Children record the average arm span in Problem 2 on journal page 256.

Examples drawn from third-grade curriculum materials in Everyday Mathematics. For more information, see <https://everydaymath.uchicago.edu/teachers/3rd-grade/>

What does the teacher have  
to know and understand?  
What is assumed?



# Student practice


Examples drawn from third-grade curriculum materials in Everyday Mathematics. For more information, see <https://everydaymath.uchicago.edu/teachers/3rd-grade/>


## Student Page


Data and Chance


### The Mean (Average)

Here are three stacks of pancakes:

4  



6  



2  






This is not fair.  
One person gets 6 pancakes and another gets 2.

There are 12 pancakes in all. We can move some pancakes to make the stacks equal. Then each stack will have 4 pancakes.

4  


4  


4  




This is fair.  
Each person gets 4 pancakes.

We say that 4 is the **mean** number of pancakes in each stack.

Here is how to find the mean:

**Step 1:** Find the total amount in all the groups.

**Step 2:** Find the amount that would be in each group if the groups were equal.

The mean is sometimes called the **average**.

Did You Know?

Average was first used as an English word around 1500. One of its meanings was "an equal share." When an expense was shared equally by a group of people, each person paid the average.

**Student Reference Book, p. 83**

# What do Serena's curriculum materials provide her?



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## In the materials?

- Clear articulation of learning goals aligned to standards
- Some instructional materials (but without annotation about their potential pitfalls and guidance for how to modify them)

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## In the materials?

- ❑ Clear articulation of learning goals aligned to standards
- ❑ Some instructional materials (but without annotation about their potential pitfalls and guidance for how to modify them)

## Not in the materials?

- ❑ Knowledge about her own students, their contexts, communities, resources, past learning
- ❑ Managing the discretionary spaces of her practice – teaching her students in her class, her classroom culture and norms, and specific moves in the moment to address the content, equity, engagement, behavior
- ❑ Explanations of the content from the perspective of content knowledge for teaching
- ❑ Insights into how students might think and how students' knowledge and experience might interact with the content
- ❑ A range of instructional activities and tasks well-coordinated to the learning goals

# Serena's teaching: Viewing focus

1. How is the instructional material shaping this teacher's lesson?
2. What is the mathematical point of this lesson, according to the teacher's guide?
3. What issues arise that the instructional material does not provide guidance for?



# Video: Serena's lesson on calculating the mean





## Discuss

1. How is the instructional material shaping this teacher's lesson?
2. What is the mathematical point of this lesson, according to the teacher's guide?
3. What issues arise that the instructional material does not provide guidance for?

Explain that children are now going to calculate another typical arm span, the **mean** arm span. Remind them that they found the mean family size in Lesson 10-6 and that **average** is another word for *mean*.

Divide the class into groups of four or five and guide children through the three steps listed below. Each child should do the computations with a calculator, compare his or her results to those of other members of the group after each step, and resolve discrepancies before going on to the next step.

1. Add all the arm-span lengths to find the class total (sum).
2. Count the stick-on notes (number of measurements).
3. Divide the total (sum) by the number of stick-on notes (number of measurements).

The result is the *mean arm span* of children in the class. If the number has decimal places, tell children they can ignore any digits after tenths. Children record the average arm span in Problem 2 on journal page 256.

Examples drawn from third-grade curriculum materials in Everyday Mathematics. For more information, see <https://everydaymath.uchicago.edu/teachers/3rd-grade/>



# What work does Serena need to be able to do to use this instructional material to enact high-quality instruction?

- Identify the main mathematical point of the lesson and the role of the activity to support the learning goals.
  - Is it about calculating an average or is it about the concept of “average” and what it means?
- Consider what her students might say or do, and what they might need support with, for example:
  - The terms “average” and “mean”: what other experiences with and knowledge of these words might the children already have?
  - The difference between averages, equal, and fairness; what the term “mean” refers to
- Develop a complete plan for teaching the lesson, including details, for example:
  - See problems with the “arm span” context and modify the task from arm span to something mathematically similar, but more useful for understanding the concept (e.g., not different amounts of candy, but different distances or lengths) and the same complexity.
  - Whether and how to use calculators
  - Planning how the discussion of the results will go: how to launch and conclude, specific questions to ask, how to use the board
- Responding to individual students about engagement, behavior, etc.



# What can instructional materials provide?

## In the materials?

- ❑ Clear articulation of learning goals aligned to standards
- ❑ Explanations of the content from the perspective of content knowledge for teaching
- ❑ Insights into how students might think and how students' knowledge and experience might interact with the content
- ❑ A range of instructional activities and tasks well-coordinated to the learning goals

## Not in the materials?

- ❑ Knowledge about your own students, their contexts, communities, resources, past learning
- ❑ Plans for teaching your students in your class, your classroom culture and norms, and specific moves in the moment to address the content, equity, engagement, behavior

# What is the work of using instructional materials?

1. Identify the core concepts and fundamental skills and practices in a unit or lesson and the most critical tasks and activities intended to support work on these concepts, skills, and practices. What is the idea and what is important about it?
2. Identify the role of other activities and tasks (e.g., review, reinforcement, challenge, supplement) and evaluate the relative importance of these and whether and how much time to devote to them.
3. Consider one's own students, their resources and ways of reasoning, and how the lesson might play out with them, and determine ways to connect, scaffold, structure the work.
4. Develop a complete plan to use for teaching the lesson in your classroom (consider children, physical environment, time factors, language, materials, pacing, assessment).



### 3. How could teachers be better supported for using and learning from curriculum materials?



# Work explicitly to support teachers to use instructional materials in teaching

1. Learning to prepare for the core content, skills, and practices and the alignment of the tasks and activities with the learning goals
2. Learning to coordinate an instructional design with your own students and context
3. Learning to use the written guidance to develop a complete plan to use for instruction

Decomposition of the practice of *Using Instructional Materials*, developed at TeachingWorks (2020)

1. Learning to prepare for the core content, skills, and practices and the alignment of the tasks and activities with the learning goals

This involves having opportunities to practice:

- a. Doing the same tasks that the children will do.
- b. Discussing the tasks.
- c. Considering how the core content, skills, and practices are aligned with the tasks.
- d. Practicing explaining, unpacking, expanding ideas and practices to connect or better address the core content, skills, and practices.



1. Learning to prepare for the core content, skills, and practices and the alignment of the tasks and activities with the learning goals

## Fractions as Numbers

### Topic D: Fractions on the

**Number Line** Students transfer their work to the number line in Topic D. They begin by using the interval from 0 to 1 as the whole. Continuing beyond the first interval, they partition, place, count, and compare fractions on the number line. (3.NF.3d)

### Topic D: Fractions on the Number Line

#### LESSONS 14-19

In Topic C, students compared unit fractions and explored the importance of specifying the whole when doing so. In Topic D, they apply their learning to the number line. Number bonds and fraction strips serve as bridges into this work. Students see intervals on the number line as wholes. They initially measure equal lengths between 0 and 1 with their fraction strips. They then work with number lines that have endpoints other than 0 and 1 or include multiple whole number intervals. This naturally transitions into comparing fractions with the same denominator, as well as fractional numbers and whole numbers on the number line. As students compare, they reason about the size of fractions and contextualize their learning within real-world applications.

Note: Examples are drawn from third-grade curriculum materials in Zearn Math. For more information, see <https://www.zearn.org/>



MISSION OVERVIEW

Grade 3, Mission 5

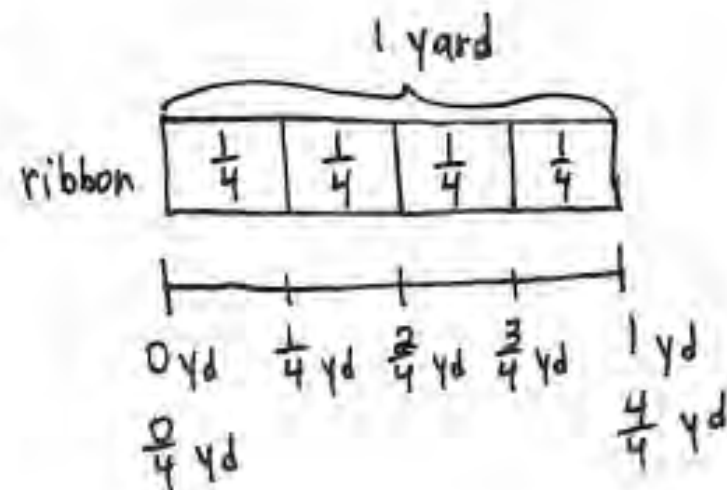
Fractions as Numbers



## LESSON 16

### Word Problem (7 min)

Hannah bought 1 yard of ribbon to wrap 4 small presents. She wants to cut the ribbon into equal parts. Draw and label a number line from 0 yards to 1 yard to show where Hannah will cut the ribbon. Label all the fractions, including 0 fourths and 4 fourths. Also, label 0 yards and 1 yard.



**Note:** This problem reviews the concept of placing fractions on a number line from Small Group Lessons 14 and 15.

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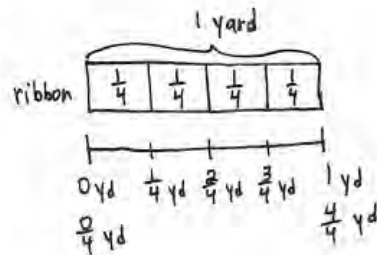
# Do the problem yourself

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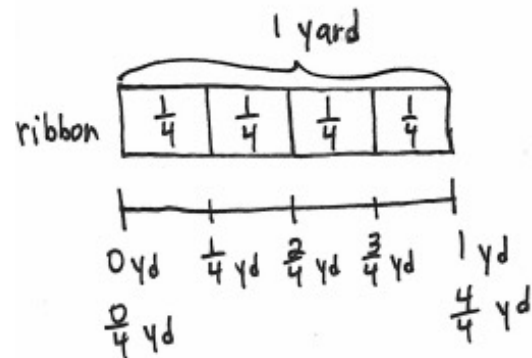
**Note:** This problem reviews the concept of placing fractions on a number line from Small Group Lessons 14 and 15.



- What is the core mathematical concept?
- What is the mathematical skill or practice?

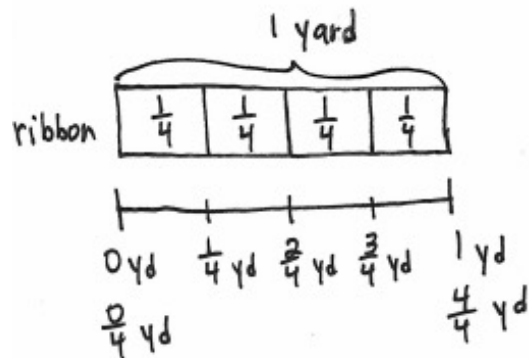
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# How does this problem relate to the core concept?



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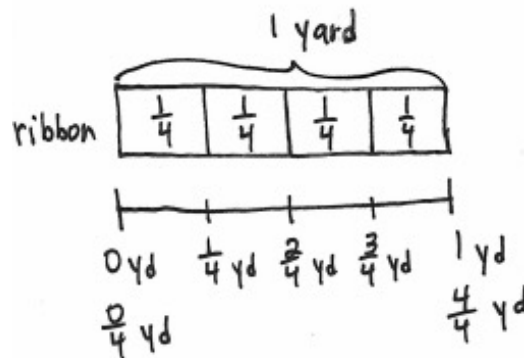
- The ribbon is supposed to model the number line.
- The whole is the yard of ribbon, which is mapped onto the interval 0 to 1 on the number line.
- Cutting the ribbon into 4 equal parts is connected to labeling the number line in fourths.



# How does this problem relate to the core concept?

What is not clear?

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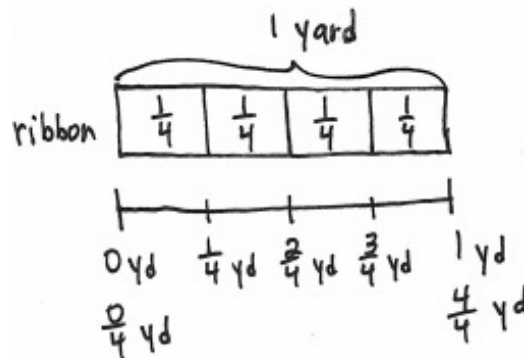




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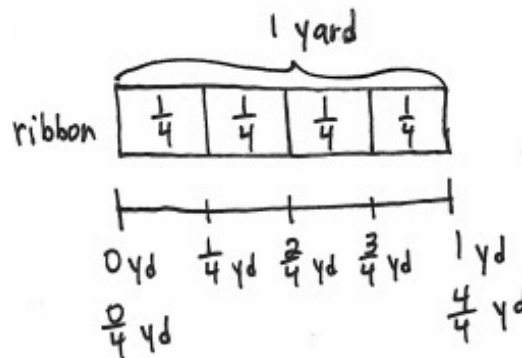


On the representation of the ribbon, each part is labeled  $\frac{1}{4}$  but on the number line, the tick marks are labeled  $\frac{1}{4}$ ,  $\frac{2}{4}$ ,  $\frac{3}{4}$ , and  $\frac{4}{4}$ .

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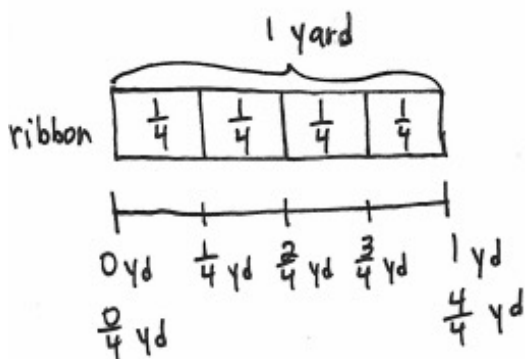
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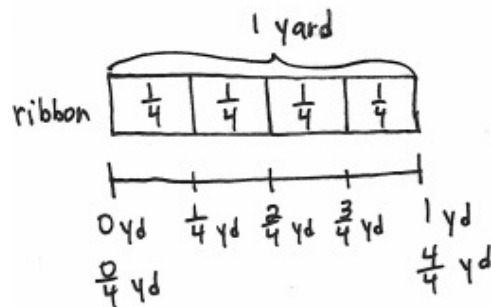


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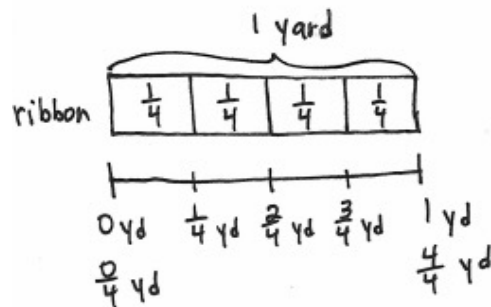
Because the ribbon is actually an area model and in area models, each equal part can be represented by  $\frac{1}{b}$ . But on the number line, the ends of each equal distance are counted from 0.

# What are the main mathematical skills or practices?



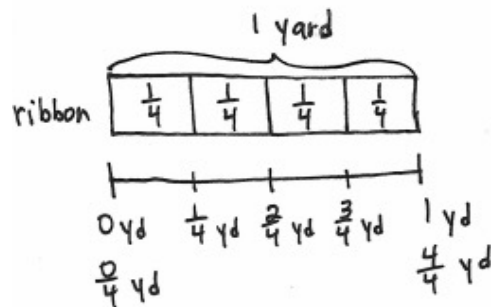
# What are the main mathematical skills or practices?

- Labeling a number line in fractions.
- Identifying one of the equal parts as  $1/b$ , the unit fraction. (3.NF.3d)
- Mapping between two representations — an area model and a number line.
- Maybe?
  - Constructing viable arguments and critiquing the reasoning of others.
  - Attending to precision.



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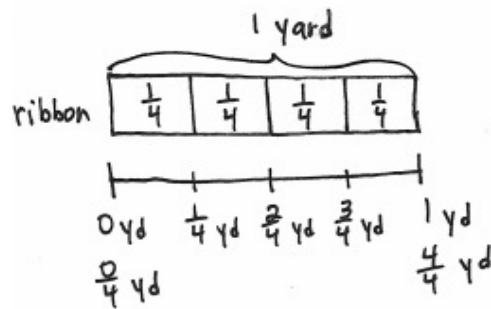
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  - Constructing viable arguments and critiquing the reasoning of others.
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I need to practice this.

- I'll try to explain how the ribbon maps on to the number line.
- Is my argument viable?
- How else might someone make that argument?



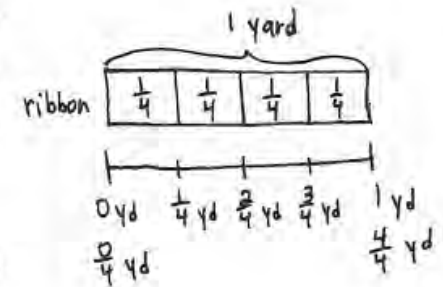
## 2. Learning to coordinate an instructional design with your own students and context

### LESSON 16

#### Word Problem (7 min)

Hannah bought 1 yard of ribbon to wrap 4 small presents. She wants to cut the ribbon into equal parts. Draw and label a number line from 0 yards to 1 yard to show where Hannah will cut the ribbon. Label all the fractions, including 0 fourths and 4 fourths. Also, label 0 yards and 1 yard.

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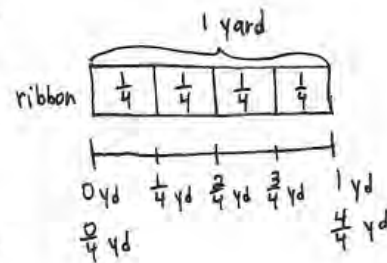
# Considering your own students and context

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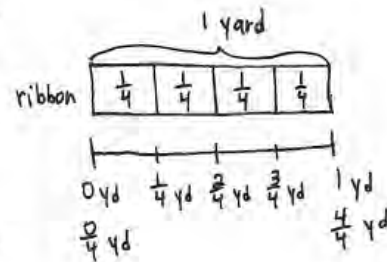
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- Anticipating what your own students might say and do
- Identifying connections to students' family and community resources
- Noticing language, tasks, or activities that could create confusion or misunderstanding
- Considering scaffolds and connections to support students
- Identifying opportunities to support students in challenging oppressive dominant narratives

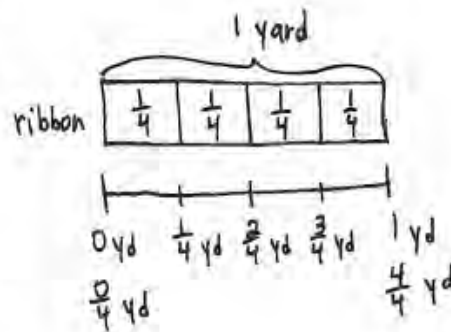
FOCUS: Notice language, tasks, or activities that might create confusion or misunderstanding.

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# Considering your students and context

## Accessibility and relevance of the lesson's ideas, examples, language

- Language: “yard”
- Mathematics: the difference between the ribbon representation and the number line
- Story context: cutting a ribbon into four equal lengths to wrap four presents
- Reasonableness and connection: Does the story make sense? Does mapping the story on to the number line make sense?
- The naming of a unit fraction  $1/b$  is not explicit here?

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## 2. Learning to coordinate an instructional design with your own students and context

This involves having opportunities to practice:

- a. Anticipating what your own students might say and do
- b. Identifying connections to students' family and community resources
- c. Noticing language, tasks, or activities that could create confusion or misunderstanding
- d. Considering scaffolds and connections to support students
- e. Identifying opportunities to support students in challenging oppressive dominant narratives



### 3. Learning to use the written guidance to develop a complete plan to use for instruction

- Plan key language to use in explaining, asking questions, labeling, etc., attending to supporting students' understanding, anticipate and avert creating misconceptions or distortions of the core content;
- Design additions or changes (additional framings, tasks, examples, exit tickets, etc.) that need to be made to scaffold learning or otherwise support learners, including additional questions or tasks that might be used if necessary, depending on how the lesson goes;
- Modify, omit, or replace contexts, examples, or other elements that reflect racial or gender bias, or other oppressive narratives, while maintaining the core content learning goals;
- Scale tasks up or down in difficulty to differentiate, reinforce, and extend instruction, being careful to maintain and support intellectual demand;
- Estimate how to distribute instructional time across the different specific parts of the lesson (e.g., whole group discussion, partner work, closing, etc.) with an eye to the main goals of the lesson and supporting one's students;
- Add details related to specific teaching moves, grouping structures, and adaptations for one's own students in a particular class;
- Create a usable form of the plan to use while teaching the lesson.

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Scale tasks up or down in difficulty to differentiate, reinforce, and extend instruction, being careful to maintain and support intellectual demand

### Option 1:

- Explain “yard” and how it is different from “backyard”
- Develop the connection to the number line, since this is the mathematical point in this series of lessons

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**Option 2:**

- Develop a new story context that is
  - mathematically the same, but where the story context makes more sense for the number line
  - careful about language

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**Option 1:**

- Explain “yard” and how it is different from “backyard”
- Develop the connection to the number line, since this is the mathematical point in this series of lessons

**Option 3:**

- Develop a student-accessible set of questions to guide the naming of fractions in  $1/b$  and multiples of  $1/b$  to align better with the standard.

**Option 2:**

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### 3. Learning to use the written guidance to develop a complete plan to use for instruction

This involves having opportunities to practice:

- Planning language to use;
- Anticipating additions or changes (additional framings, tasks, examples, exit tickets, etc.) that need to be made to scaffold learning or otherwise support learners, including additional questions or tasks that might be used if necessary, depending on how the lesson goes;
- Modifying, omitting, or replacing contexts, examples, or other elements that reflect racial or gender bias, or other oppressive narratives, while maintaining the core content learning goals;
- Scaling tasks up or down in difficulty to differentiate, reinforce, and extend instruction, being careful to maintain and support intellectual demand;
- Estimating how to distribute instructional time across the different specific parts of the lesson with an eye to the main goals of the lesson and supporting one's students;
- Adding details related to specific teaching moves, grouping structures, and adaptations for one's own students in a particular class;
- Creating a usable form of the plan to use while teaching the lesson.



# Conclusion:

(How) Could instructional materials improve mathematics teaching and learning?



# Take-aways

1. Instructional materials can provide teachers with support for teaching in ways that are coordinated to standards and sensitive to their students and contexts.
2. But instructional materials cannot do it all—know the students, recognize and draw on their contexts and communities, be sensitive to one’s setting, directly address micro-moments permeated with issues of racism, sexism, and other forms of oppression.
3. There are important things to learn in order to use (any) instructional materials with care. Left to chance, even high-quality curriculum cannot teach.
4. Teacher preparation and professional development can provide teachers focused opportunities to learn to use instructional materials wisely.
5. High-quality instructional materials are ones that would position teaching and teachers as central to enactment.



# Implications for the design of instructional materials

- Design for enactment, not implementation
- Position the teacher as key audience for the material
  - Explaining the content from a mathematical knowledge for teaching perspective
  - Build guidance aligned to be considerate of real understanding of the work of teaching (volume, time, etc.)
  - Support for pressing students' thinking and for responding
- Provide useful and culturally sensitive materials and tools for connecting with families

# Implications for professional learning

Structured opportunities in teacher preparation and professional development to:

1. Identify and unpack the specific content goals and to revisit and learn the content deeply enough for teaching.
2. Examine the correspondence of the tasks and activities with the content goals.
3. Consider one's own students and contexts and prepare supports or revisions to the lesson without compromising the content goals or distorting the content.
4. Develop a usable teaching plan for the lesson with one's own students in one's own classroom.
5. Build the mindset, habits, and skills to do this independently.

\*TeachingWorks has developed what is involved in the work of using instructional materials, including a decomposition of the practice, materials, and learning opportunities.

# Implications for those who support teachers

Coaches, teacher educators, master teachers, school leaders, professional developers need:

- Tools to support focused work with teachers
  - TeachingWorks has developed materials, tools, and professional learning for people in these roles
- Opportunities for professional learning



Save the date for our upcoming programs  
Learn more at **teachingworks.org**



1

**June 2023 in Ann Arbor, MI**  
Virtual Participation Available

### **Content Methods Laboratory Classes**

Examine the teacher education content methods courses under a microscope, while honing your skills at designing practice-based teacher education.



2

**August 2023 in Ann Arbor, MI**

### **Introduction to Practice-Based Teacher Education Workshop**

Familiarize yourself with the basic concepts and tools of practice-based teacher education and approaches to teaching instructional practice.



3

**October 2023 in Ann Arbor, MI**

### **Coaching Institute**

Unpack and practice the work of coaching and supporting beginning teachers to build your instructional skill in a supportive and dynamic environment.

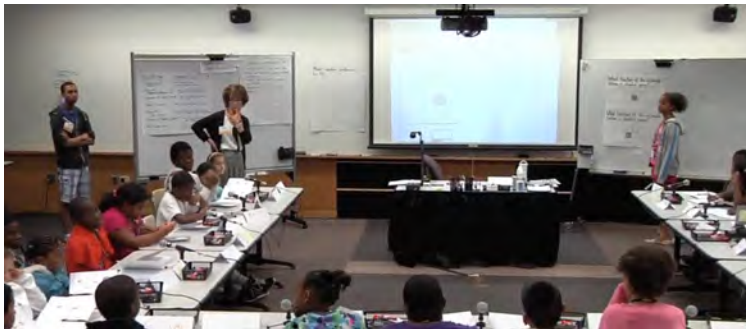


And: Information coming later this month at [teachingworks.org](https://teachingworks.org)

## 2023 Elementary Mathematics Laboratory July 24 – 28, 2023 • in southeast Michigan

*Now in its 17<sup>th</sup> year!*

- Watch a live elementary class
- Engage in professional development on a topic related to your own role and work





# Returning to our three premises that framed our work today

1. Mathematics education improvement is not a new project. Let's look back—and use—what we have learned from past efforts.
2. Instructional materials are essential for good teaching, but they cannot be a panacea. They can be a partner in improving mathematics learning.
3. Enactment is different from “implementation” and demands particular support.

# THANK YOU!



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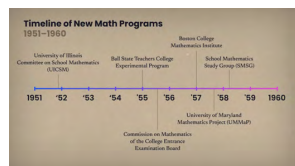


Image on slide 7:  
A timeline of new math reform organisations in the 1950s, from “What Happened to ‘New Math’?” by Asher Isbrucker, *Medium*, April 20, 2021.  
Retrieved from <https://medium.com/age-of-awareness/what-happened-to-new-math-eeb8522fc695>

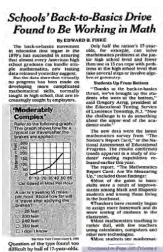


Image on slide 7:  
Screenshot of “School’s Back-to-Basics Drive Found to be Working in Math,” by Edward D. Fiske, *New York Times*, June 8, 1988.  
Retrieved from <https://www.nytimes.com/1988/06/08/us/schools-back-to-basics-drive-found-to-be-working-in-math.html>

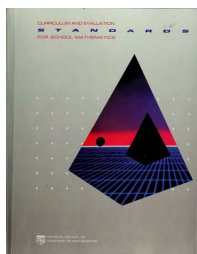


Image on slide 7:  
Cover of *Curriculum and Evaluation Standards for School Mathematics*, by the National Council of Teachers of Mathematics Commission on Standards for School Mathematics, 1989.  
Retrieved from <https://archive.org/details/curriculumevalua00nati/mode/2up>



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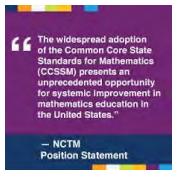


Image on slide 7:  
Graphic from "Supporting the Common Core State Standards for Mathematics," by the National Council of Teachers of Mathematics, August 2013.  
Retrieved from <https://www.nctm.org/ccssmposition/>

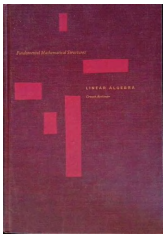


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Cover of *Linear Algebra*, 1965.  
Retrieved from <https://www.amazon.com/Linear-Algebra-Fundamental-Mathematical-Structures/dp/B002NB51BM>

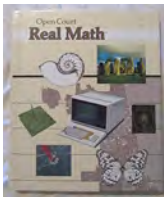


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Cover of *Open Court Real Math, Grade 7*, 1991.  
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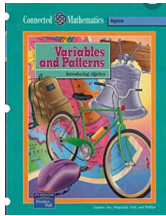


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Cover of *Connected Mathematics Variables and Patterns*, 2003.  
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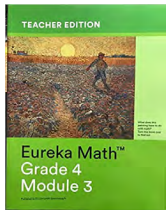


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Cover of *Eureka Math, A Story of Units: Grade 4, Module 3*, 2015.  
Retrieved from <https://www.amazon.com/Eureka-Math-Grade-Module-Teachers/dp/1632553724>

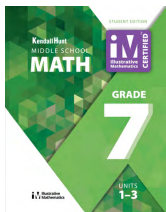


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Cover of *Illustrative Mathematics: Grade 7*, 2019.  
Retrieved from <https://k12.kendallhunt.com/product/illustrative-mathematics-grade-7-student-edition-set>



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## U.S. STUDY SHOWS PUPIL ACHIEVEMENT AT LEVEL OF 1970

American elementary and secondary school pupils have made some educational progress in recent years, but they are only now reaching the achievement levels of students in 1970, according to a major Federal report issued today, which added that students are unprepared for the complex and demanding world of the 21st century. "Today's children seem to know about as much math and about as much science and read about as well as their parents did at that age about 20 years ago," Education Secretary Lamar Alexander said of the report, "Trends in Academic Progress."

Image on slide 9:

Screenshot from "U.S. study shows pupil achievement at level of 1970," by Karen De Witt, *New York Times*, October 1, 1991.

Retrieved from <https://www.nytimes.com/1991/10/01/us/us-study-shows-pupil-achievement-at-level-of-1970.html>

## Long-Term NAEP Scores for 13-Year-Olds Drop for First Time Since Testing Began in 1970s — 'A Matter for National Concern,' Experts Say

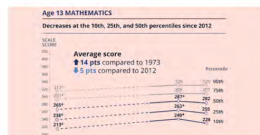


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Screenshot from "Long-Term NAEP Scores for 13-Year-Olds Drop for First Time Since Testing Began in 1970s — 'A Matter for National Concern,' Experts Say," by Kevin Mahnken, *The74*, October 14, 2021.

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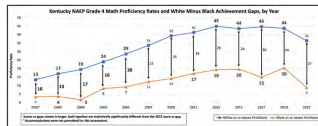


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Graphic from "What happened to Kentucky's NAEP achievement gaps for math?," by Richard Innes, Bluegrass Institute, October 30, 2022.

Retrieved from <https://bipps.org/blog/what-happened-to-kentuckys-naep-achievement-gaps-for-math>

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Photo from “What do teachers need this school year? Laura McClure from TED-Ed resolved to find out,” by Laura McClure, TED, September 7, 2016.

Retrieved from <https://ideas.ted.com/how-to-help-a-teacher-out/>

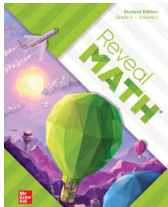


Image on slide 11:

Cover of *Reveal Math: Grade 4*, 2022.

Retrieved from <https://www.mheducation.com/prek-12/product/reveal-math-student-edition-grade-4-volume-1-mcgraw-hill/9780076659357.html>



Image on slide 11:

Graphic from “Teacher Evaluation: Why it Matters and How We Can Do Better,” by Sheila B. Robinson, Frontline Education.

Retrieved from <https://www.frontlineeducation.com/teacher-evaluation/>



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Photo from “Manipulatives Help Kids Understand Math Concepts” from Mathnasium  
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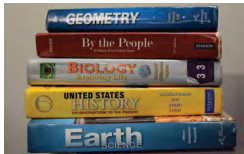


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Photo of text books from Ben Davis High School  
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