

Using Discretionary Spaces in Teaching to Advance Students' Mathematical Flourishing

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Baltimore City Public Schools

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Teaching is powerful.



Understanding the power of the work of teaching

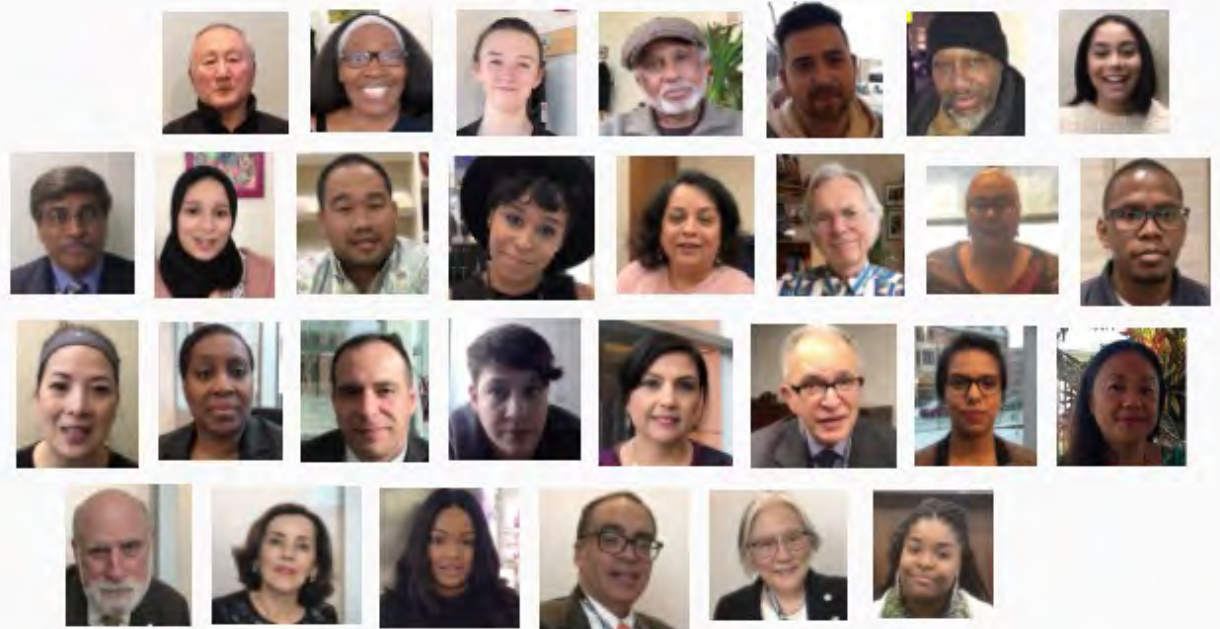
How is teaching powerful?

What are “discretionary spaces” and what makes them so powerful?

How can we better leverage the discretionary spaces in our work?



1. How is teaching powerful?



Video: Is there a teacher who had a significant impact on you?



6



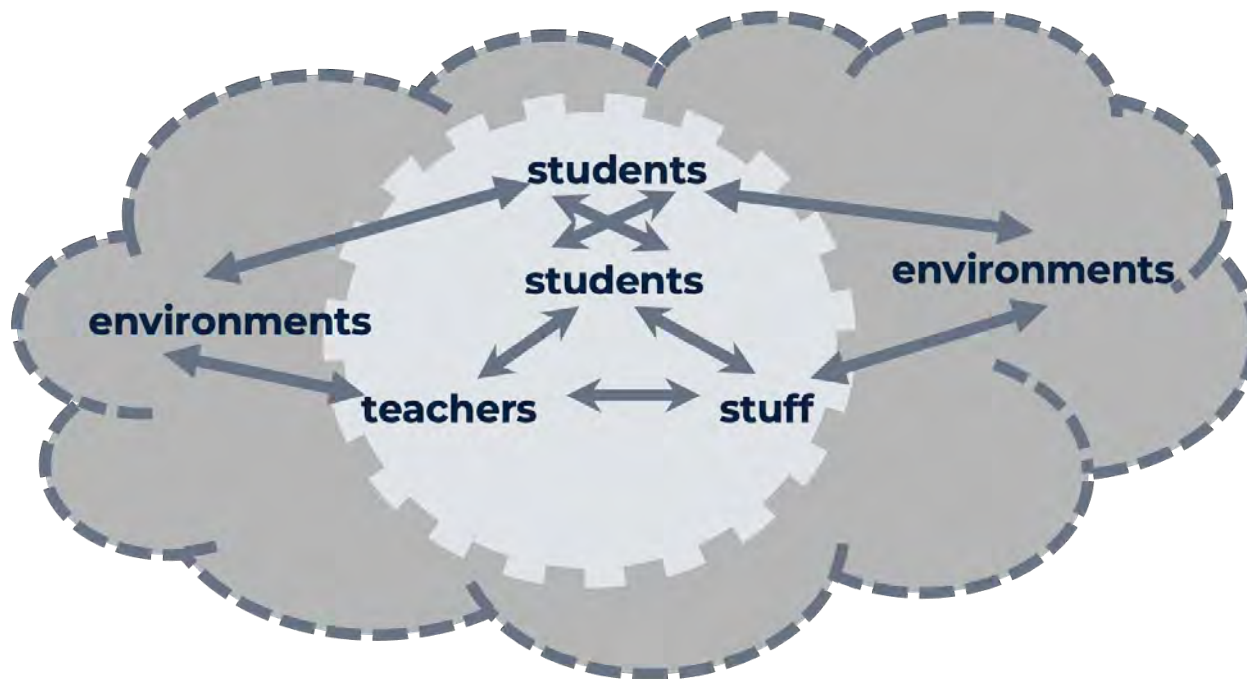
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Why does it matter when we don't pay attention to the actual work of teaching?

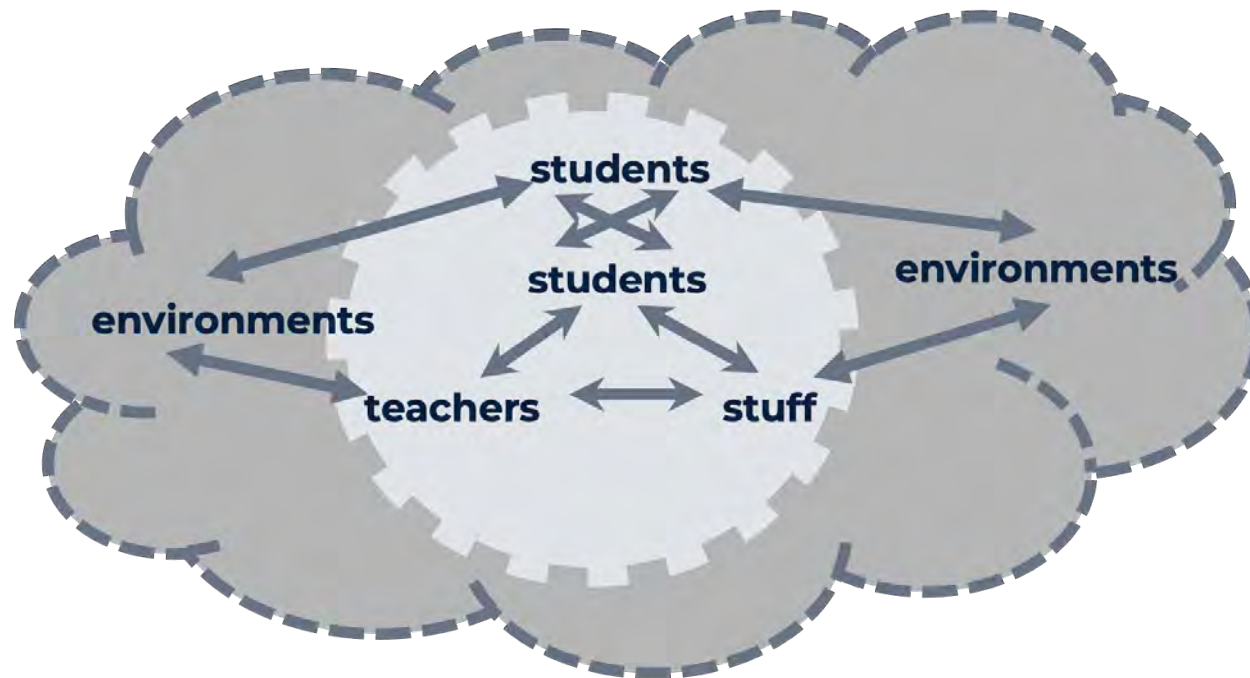


We fail to connect the dots.

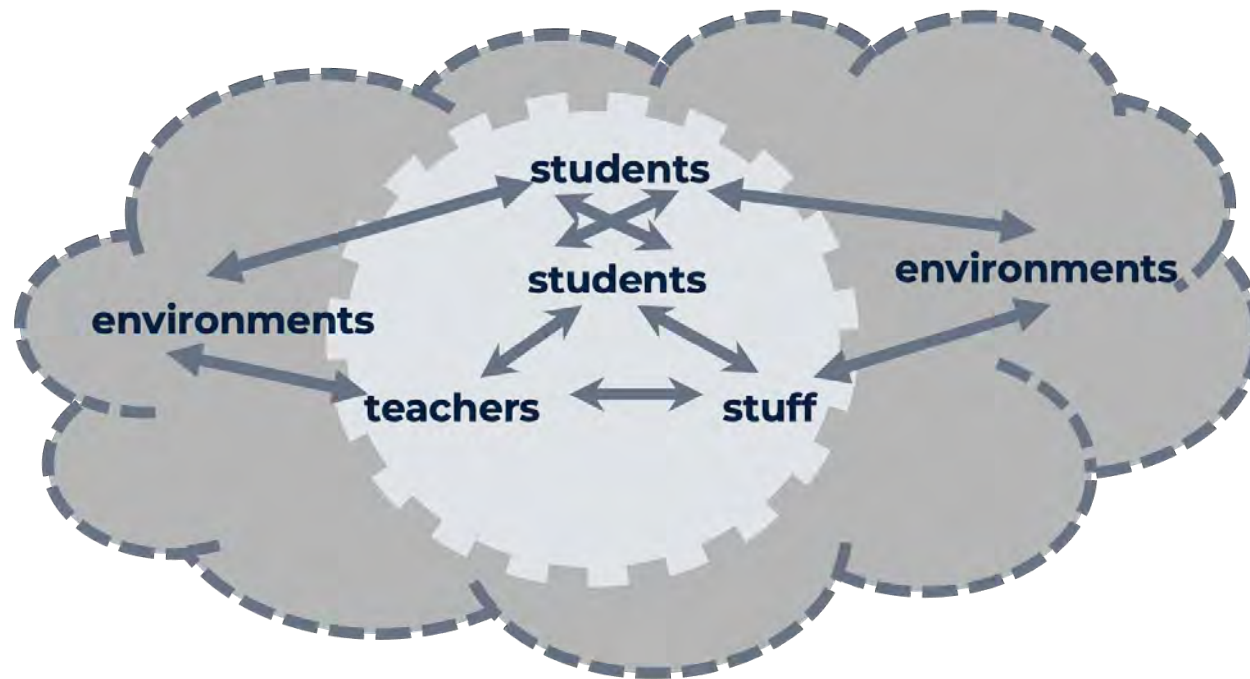
And that means that our strategies often miss the mark of helping students flourish—and reproduce patterns of inequity and injustice.



1. Teaching and learning are constructed interactively and are interpretive.



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2. They take place within broad historical and socio-political environments.



How do macro-level factors interact with practice?



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Histories, policies, institutional systems

(e.g., institutional racism, history of intelligence as the property of white people, anti-Black “scientific” racism; legacies of violence against Black and Indigenous people)



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Taken-for-granted practices and norms

(e.g., teachers’ judgments of students, who is positioned as “smart” or “struggling”; disproportional punishment and assignment to special ed and gifted programs)

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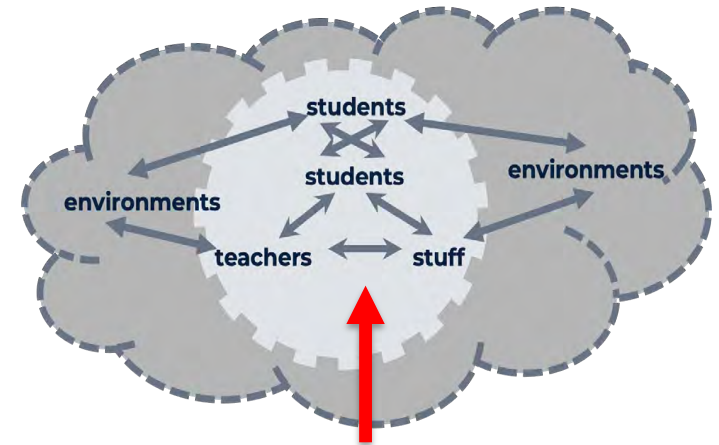
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(e.g., teachers’ judgments of students, who is positioned as “smart” or “struggling”; disproportional punishment and assignment to special ed and gifted programs)



The arrows run both ways.

The environments in which we are all steeped shape what we take for granted and do, but learning these as normal also reinforces their persistence.





NOTICE:
VOTERS ARE REQUIRED TO WEAR MASKS,
PRACTICE SOCIAL DISTANCING,
AND BE MINDFUL OF OTHER PEOPLE
IN YOUR COMMUNITY
6 FEET





1. **Teaching is powerful.** When it is done with care and judgment, students can thrive—learn content, develop positive identities, learn to value others and work collectively.
2. Teaching also involves **enormous discretion.**
3. How that discretion is exercised can either reinforce racialized and oppressive patterns of social, personal, and epistemic injustice and harm, or it can **disrupt these patterns.**



How can we leverage the power of teaching?

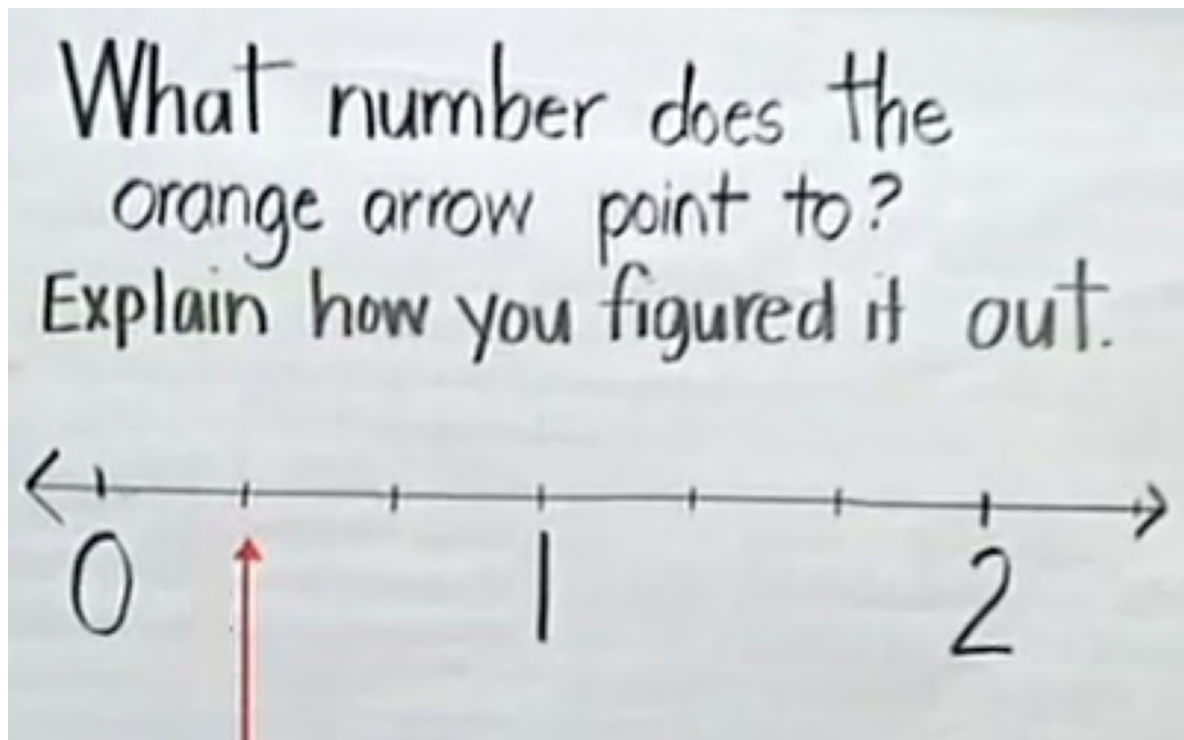


Fractions as numbers: The instructional goal for student understanding

3.NF.2.a

Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.







Aniyah



Toni

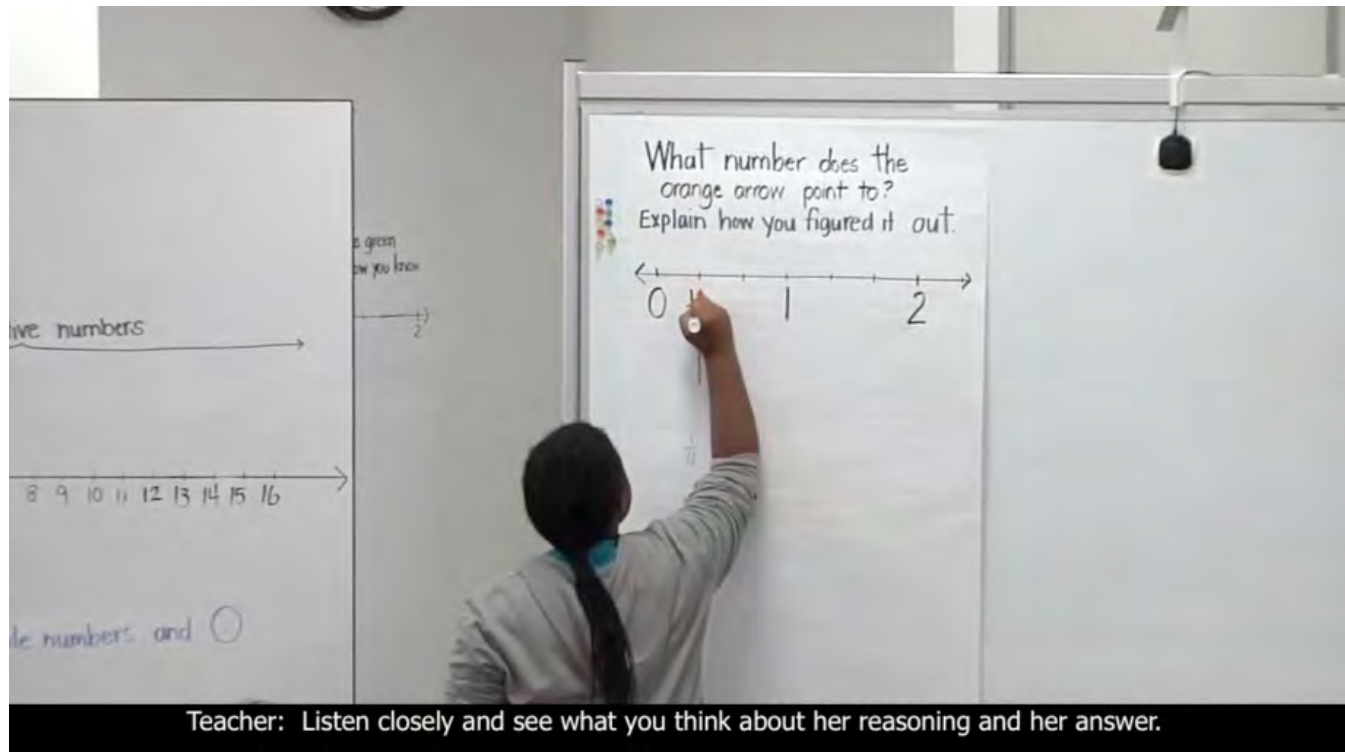


Viewing focus

What do you see about each girl's mathematical knowledge and skill?



Video: Aniyah and Toni



This video and additional supporting materials are available online [here](#).



Discuss

What do you see about each girl's mathematical knowledge and skill?



What would
happen
next?

What are the
possible
results?



In the next moment, what is likely to happen?

NORMALIZED NEXT MOVES

RESULTS



In the next moment, what is likely to happen?

NORMALIZED NEXT MOVES

- “Can someone help Aniyah out and show what we call the whole on the number line?”

RESULTS

- Aniyah is excluded and her mathematical contributions are sidelined.



In the next moment, what is likely to happen?

NORMALIZED NEXT MOVES

- “Can someone help Aniyah out and show what we call the whole on the number line?”
- “Great, Aniyah, almost! But remember that the whole is from 0 to 1.”

RESULTS

- Aniyah is excluded and her mathematical contributions are sidelined.
- Aniyah’s answer is signaled to be incorrect and she is positioned as not having contributed to the work.



In the next moment, what is likely to happen?

NORMALIZED NEXT MOVES

- “Can someone help Aniyah out and show what we call the whole on the number line?”
- “Great, Aniyah, almost! But remember that the whole is from 0 to 1.”
- “Thumbs up if you agree with Aniyah; thumbs down if you disagree.”

RESULTS

- Aniyah is excluded and her mathematical contributions are sidelined.
- Aniyah’s answer is signaled to be incorrect and she is positioned as not having contributed to the work.
- Aniyah’s solution is “voted” on by her classmates.



What else is likely?

NORMALIZED NEXT MOVES

RESULTS



What else is likely?

NORMALIZED NEXT MOVES

- “Toni, when you’re ready to participate appropriately by not playing with your hair and laughing, and have a question to ask, I will come back to you.”

RESULTS

- Toni is publicly excluded from the discussion.



What else is likely?

NORMALIZED NEXT MOVES

- “Toni, when you’re ready to participate appropriately by not playing with your hair and laughing, and have a question to ask, I will come back to you.”
- “You need to be a better listener, Toni. Aniyah already explained why she picked one-seventh. Who else has a real question for Aniyah?”

RESULTS

- Toni is publicly excluded from the discussion.
- Toni is judged to not be listening, her question is judged as not good, and she is excluded from the discussion.



What else is likely?

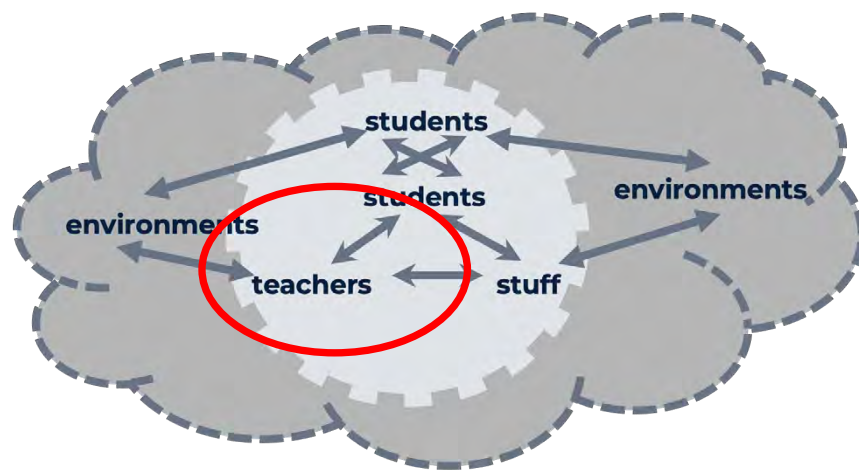
NORMALIZED NEXT MOVES

- “Toni, when you’re ready to participate appropriately by not playing with your hair and laughing, and have a question to ask, I will come back to you.”
- “You need to be a better listener, Toni. Aniyah already explained why she picked one-seventh. Who else has a real question for Aniyah?”
- “In this classroom, we are respectful of one another. When you are ready to be respectful, you can rejoin the discussion, Toni.”

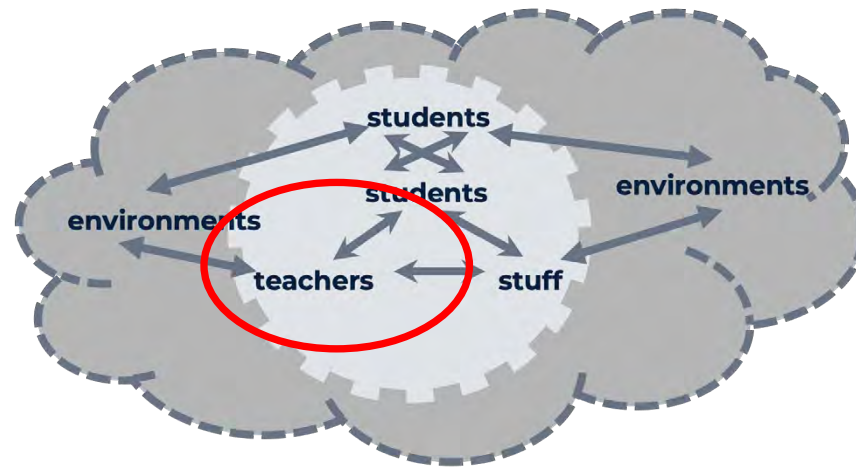
RESULTS

- Toni is publicly excluded from the discussion.
- Toni is judged to not be listening, her question is judged as not good, and she is excluded from the discussion.
- Toni is publicly named and shamed as “disrespectful,” rebuked, and her role in advancing the mathematics is sidelined.

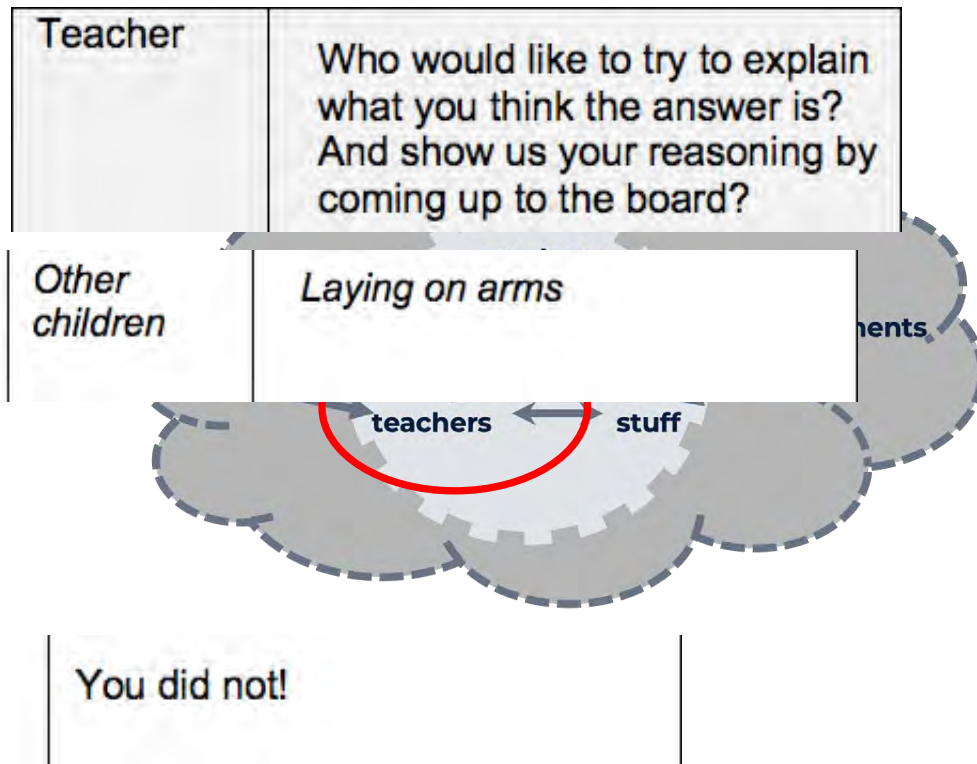




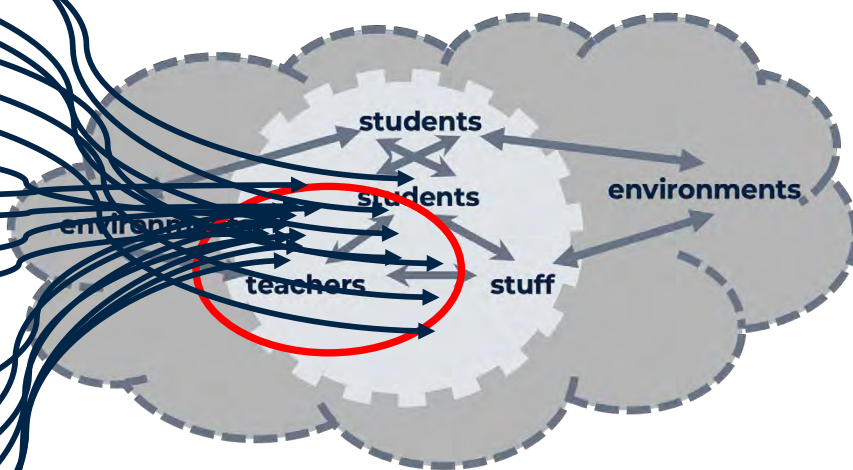
Speaker	Talk	Discretionary space
Teacher	Who would like to try to explain what you think the answer is? And show us your reasoning by coming up to the board? Who'd like to come up to the board and try to tell- And you know, it might not be right. That's okay because we're learning something new. I'd like someone to come up and sort of be the teacher and explain how you are thinking about it. Who'd like to try that this morning?	1. Deciding when to open whole-group discussion 2. Deciding what to do to launch discussion 3. Framing the expectation for presenting 4. Framing of what it "coming to the board" entails
Teacher	Okay, Anyiah?	5. Selecting a student to present
Toni	Playing with hair	6. Deciding whether to comment
Other children	Laying on arms	7. Deciding whether to comment
Teacher	When someone's presenting at the board, what should you be doing?	8. Setting norms for what to do when a student is presenting
Students in chorus	Looking at them.	
Teacher	Looking at that person.	9. Responding to students
Teacher	Uh-huh?	10. Taking up an individual student question
Anyiah	You want me to write it?	
Teacher	You're trying to mark what you think this number is and explain how you figured it out.	11. Clarifying task
Teacher	Listen closely and see what you think about her reasoning and her answer. (Anyiah writes $\frac{1}{7}$ by the orange line).	12. Setting task for the other students
Anyiah	I put one-seventh because there's-	
Toni	Did she say one-seventh?	13. Responding to student
Anyiah	(turns to Toni) Yeah... (continues to class) Because there's seven equal parts, like one, two, three, four, five, six, and then seven. (Uses her fingers to count the parts on the number line).	
Teacher	Before you agree or disagree, I want you to ask questions if there's something you don't understand about what she did. No agreeing and disagreeing. Just. All you can do right now is ask Anyiah questions. Who has a question for her?	14. Setting task for responding to student explanation
Teacher	Okay, Toni, what's your question for her?	15. Selecting student to speak
Dante	You did not!	16. Responding to student speaking across room
Toni	Why did- (laughs at another student who says something to her from across the room)	17. Responding to student laughing
Teacher	Go ahead, it's your turn.	17. Responding to student laughing
Toni	Why did you pick one-seventh?	
Dante	You did not!	18. Responding to student speaking across room
Teacher	Let's listen to her answer now. That was a very good question.	19. Setting task for class 20. Responding to student



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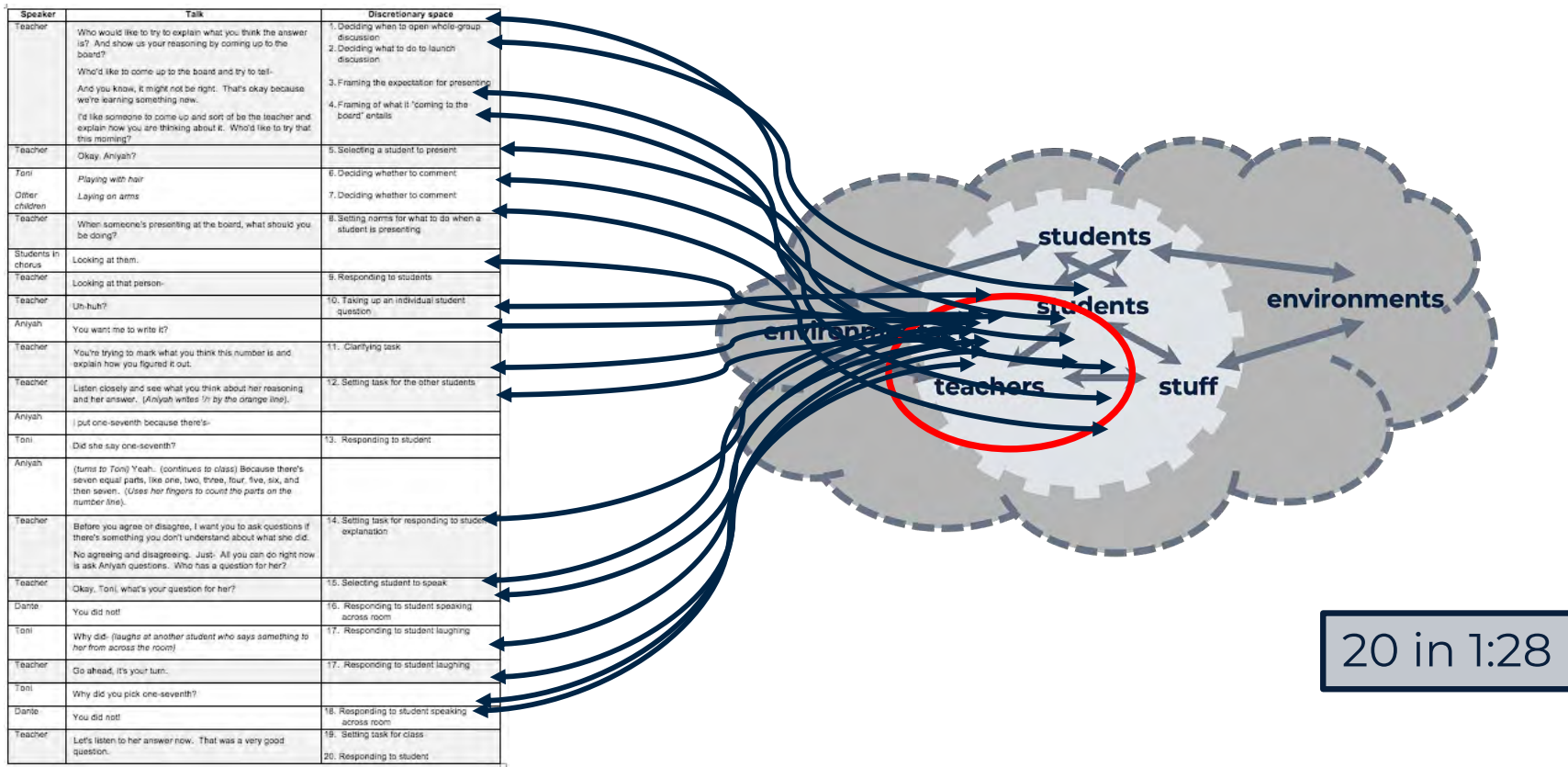
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20 in 1:28



Teaching is dense with “discretionary spaces”



20 in 1:28

Video: Aniyah and Toni



This video and additional supporting materials are available online [here](#).

Using discretionary spaces with care



- Interpreting Toni as asking a real question that she means.
- Hearing Toni's question as central to advancing the mathematical content.
- Reinforcing her mathematical identity, not choosing to read her body as disruptive.

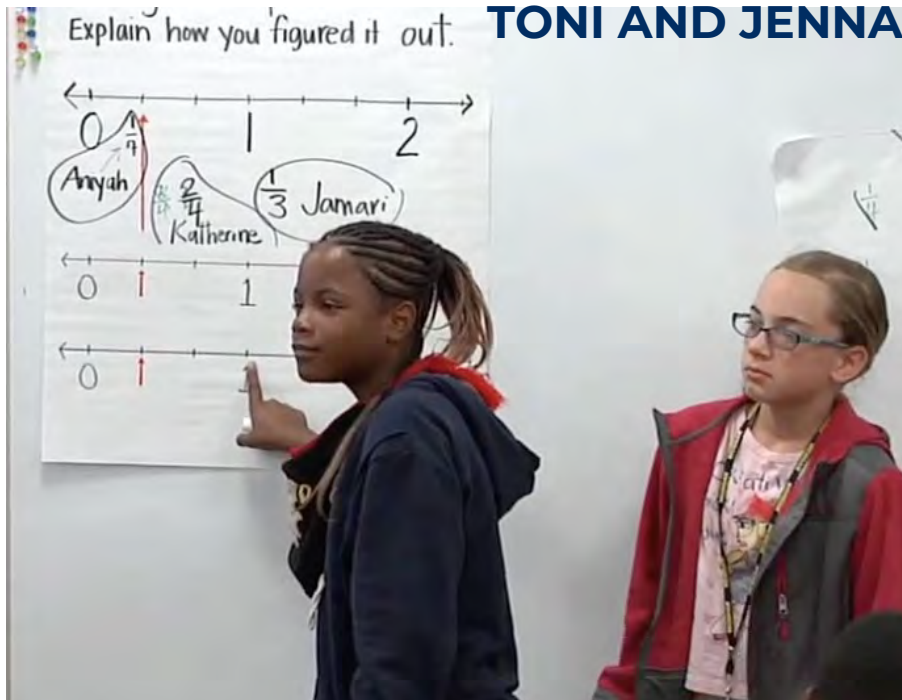


- Other children hear Aniyah as getting the discussion going, and Toni as asking an important mathematical question.
- Aniyah and Toni are both positioned as contributing to the discussion.
- Children see a teacher attending to Black girls as mathematical thinkers and contributors to collective work.



- Interpreting Aniyah as competent to answer questions about her ideas.
- Hearing Aniyah's explanation as central to advancing the mathematical content.
- Reinforcing her mathematical identity, not choosing to read her body as struggling.

14 minutes after where we stopped

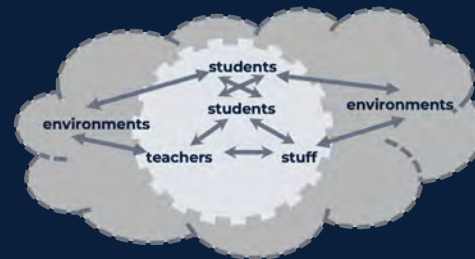


ANIYAH

I did well on my goal today because my goal was to to share my ideas with the class and I did I went up to the board and share my idea with the class on Fractions.

AND: On an “exit ticket,” 28 of the children were able to correctly identify a different fraction ($\frac{2}{5}$) on the number line.

What does it take to disrupt the patterns that marginalize and reinforce racism?

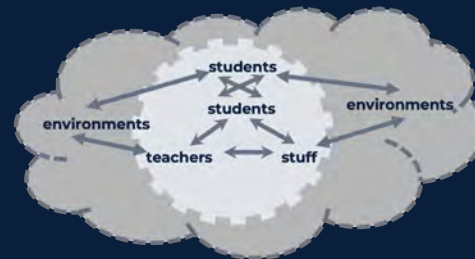


What does it take to disrupt the patterns that marginalize and reinforce racism?



- Seeing Aniyah's explanation and Toni's question as key to the class's work

Gholson & Martin, 2014; Langer-Osuna, 2015, 2017; Leonard & Martin, 2013; Martin, 2012, 2015



What does it take to disrupt the patterns that marginalize and reinforce racism?



- Seeing Aniyah's explanation and Toni's question as key to the class's work
- Taking as axiomatic the brilliance of Black children, and thus Toni and Aniyah

Gholson & Martin, 2014; Langer-Osuna, 2015, 2017; Leonard & Martin, 2013; Martin, 2012, 2015



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Knowing and using mathematics in teaching (MKT)

Interpreting Toni as asking a mathematical question that she means

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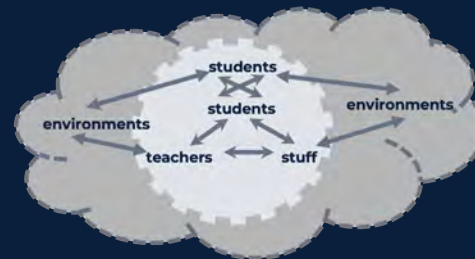


- Seeing Aniyah's explanation and Toni's question as key to the class's work
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- ... AND having something different to DO

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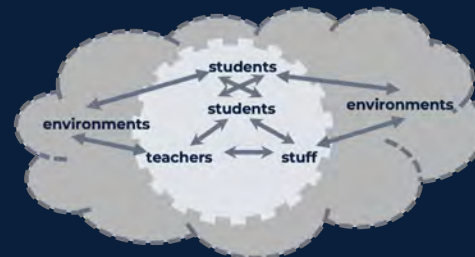


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Knowing and using mathematics in teaching (MKT)

Interpreting Toni as asking a mathematical question that she means

Having a repertoire of practices that can be adapted and used in contexts



Gholson & Martin, 2014; Langer-Osuna, 2015, 2017; Leonard & Martin, 2013; Martin, 2012, 2015



Discretionary spaces lie in the many, many moment-to-moment moves, uses of language, and decisions that are inevitable in practice.

These are both crucial resources for supporting students and also where harm happens.

What would it take to learn to use the discretionary spaces in teaching in ways that advance students' mathematical flourishing?



Using the power of discretionary spaces to support BCPS learners

Where are high-leverage discretionary spaces in teaching mathematics that matter for students' mathematical flourishing?

Where are the discretionary spaces in YOUR work that matter for students' mathematical flourishing?



Where are high-leverage discretionary spaces in **teaching mathematics** that matter for students' mathematical flourishing?



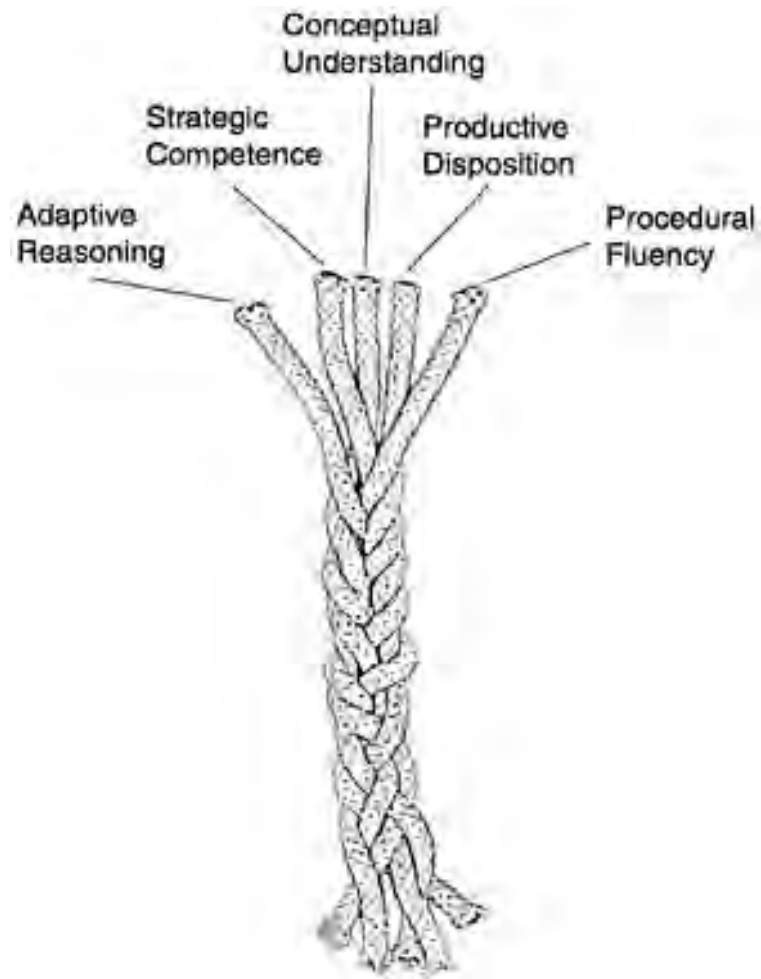
1. Developing understanding of core concepts, procedural fluency, mathematical reasoning, and automaticity
2. Acknowledging competence
3. Engaging students in explanation and reasoning
4. Using language to explain to students

Inherent discretionary spaces in teaching



1. Developing understanding of core concepts and procedures, mathematical reasoning, and automaticity





**You have
1 minute.**

Multiplication of Decimals II—Round 1

Directions: Determine the products of the decimals.

1.	4.5×3	
2.	7.2×8	
3.	9.4×6	
4.	10.2×7	
5.	8.3×4	
6.	5.8×2	
7.	7.1×9	
8.	5.9×10	
9.	3.4×3	
10.	3.2×4	
11.	6×2.8	
12.	9.7×3	
13.	8×10.2	
14.	4×8.9	
15.	3.9×7	
16.	6×5.5	
17.	1.8×8	
18.	9×2.3	



Reflect:

1. Did you enjoy doing that?
2. Did you feel competent?

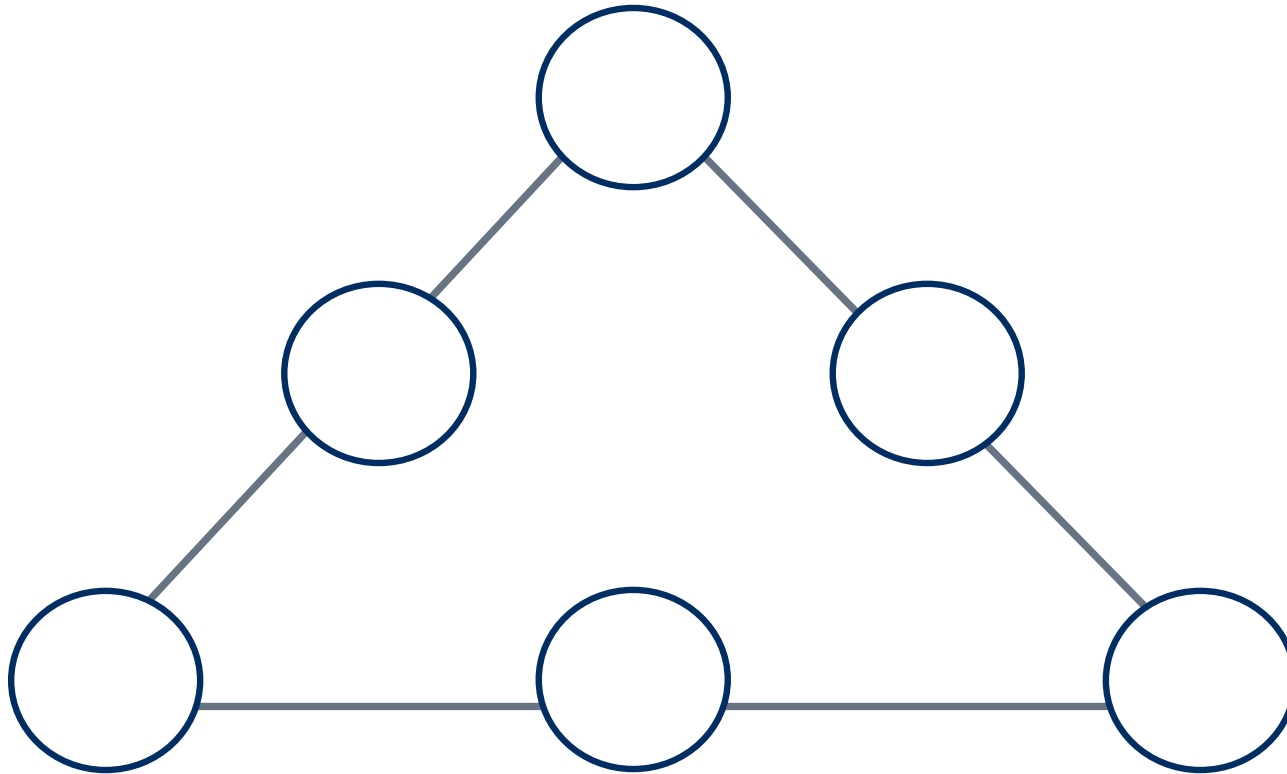


The 8's Problem

Write down as many 8's as you want in a row. Insert plus (+) signs between them so that the expression equals 1000.

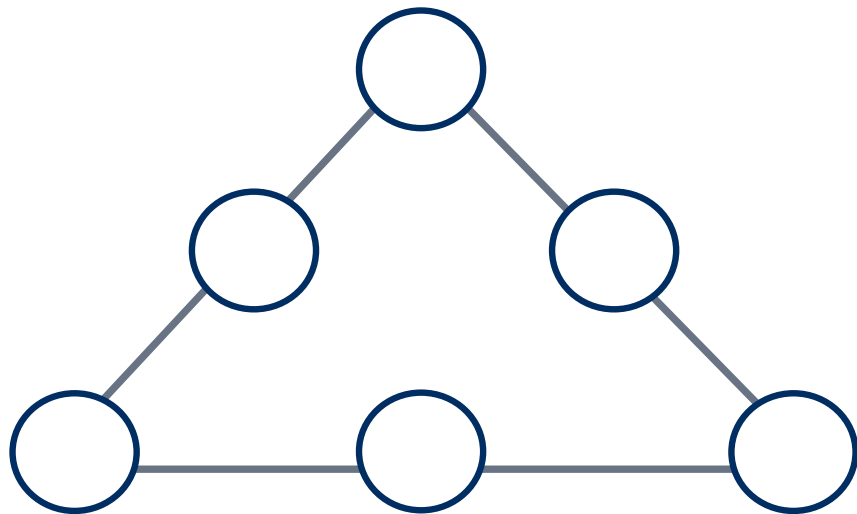
(You can put 8s together, e.g., 88.)

How many ways can you do this? Prove that you have found them all.



Place the numbers 1, 2, 3, 4, 5, and 6 in the circles, using each number exactly once.

Place them so that when you add up the values in the three circles on each of three sides of the triangle, they total the same sum.



How many different solutions are there? Prove that you have found them all.

Compare (two) tasks.

1. What mathematical knowledge and skills did each require?
2. What sorts of practice did each offer?
3. Which was most interesting? challenging?

Sprint
Multiplying
Decimals

8's Problem

Magic Triangle

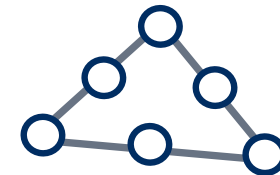
Multiplication of Decimals, 8th-Round 1

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8.	5.9×10	
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10.	3.2×4	
11.	6×2.8	
12.	9.7×3	
13.	8×10.2	
14.	4×8.9	
15.	3.9×7	
16.	6×5.5	
17.	1.8×8	
18.	9×2.3	
19.	3.5×4.1	
20.	9.3×1.7	
21.	10.4×7.6	
22.	2.7×8.3	
23.	1.8×7.8	
24.	7.5×10.1	
25.	7.2×6.3	
26.	1.9×8.3	
27.	9.8×5.1	
28.	18.2×12	
29.	13.4×22	
30.	92.3×45	
31.	86.1×16	
32.	29.7×8.2	
33.	56.8×9.5	
34.	110.3×20.2	
35.	256.6×54.9	
36.	312.8×16.5	

Number Correct: _____

The 8's Problem



Discretionary space #1:

Developing understanding of core concepts and procedures, mathematical reasoning, and automaticity

Identify, develop, and construct alternative resources for combining these key mathematical goals

2. Acknowledging competence



Discretionary space #2: Acknowledging competence

Supporting learning by how
teachers signal who and what is
seen as mathematically competent





“Positioning” theory

A theory from social psychology, originated by Bronwyn Davies and Rom Harré

Focuses on how discursive and other moves in social interactions and situations set up how people are seen, interpreted, and how people experience and feel themselves to be seen interpreted, and how they experience their sense of self in that context.

Davies, B. & Harré, R. (1990). Positioning: The discursive production of selves. *The Journal of Social Behavior*, 20(1) 43–63.



Acknowledging competence

Deliberately deploying the power of teaching to:

1. Broaden and label what being competent (“smart”) in mathematics means
2. Intervene to position who (and what) is seen as competent in mathematics
3. Support positive individual identities and self-positioning

Sources: E. Cohen and R. Lotan, complex instruction; J. Boaler’s work; *Smarter Together: Collaboration and Equity in the Elementary Mathematics Classroom* (Featherstone, Crespo, et al., 2011)



What does “acknowledging competence” require?

1. Having a broad understanding of mathematics oneself
2. Using the concept of “positioning” to intervene to influence who (and what) is seen as competent
3. Strategically making these moves in authentic and well-timed ways; having ways to do that that are sensitive



Distinguishing acknowledging competence from praise

Praise

- “Good job!”
- “You’re working so well today.”
- “Nice work!”
- “I am proud of you.”
- “You’re working like such good mathematicians.”
- “You made so much progress on the problems today.”

Praise – verbal feedback with the purpose of evaluating what a student says or does

Acknowledging competence – intentional identifying, naming, and highlighting specific mathematical or learning competencies of what a student says or does

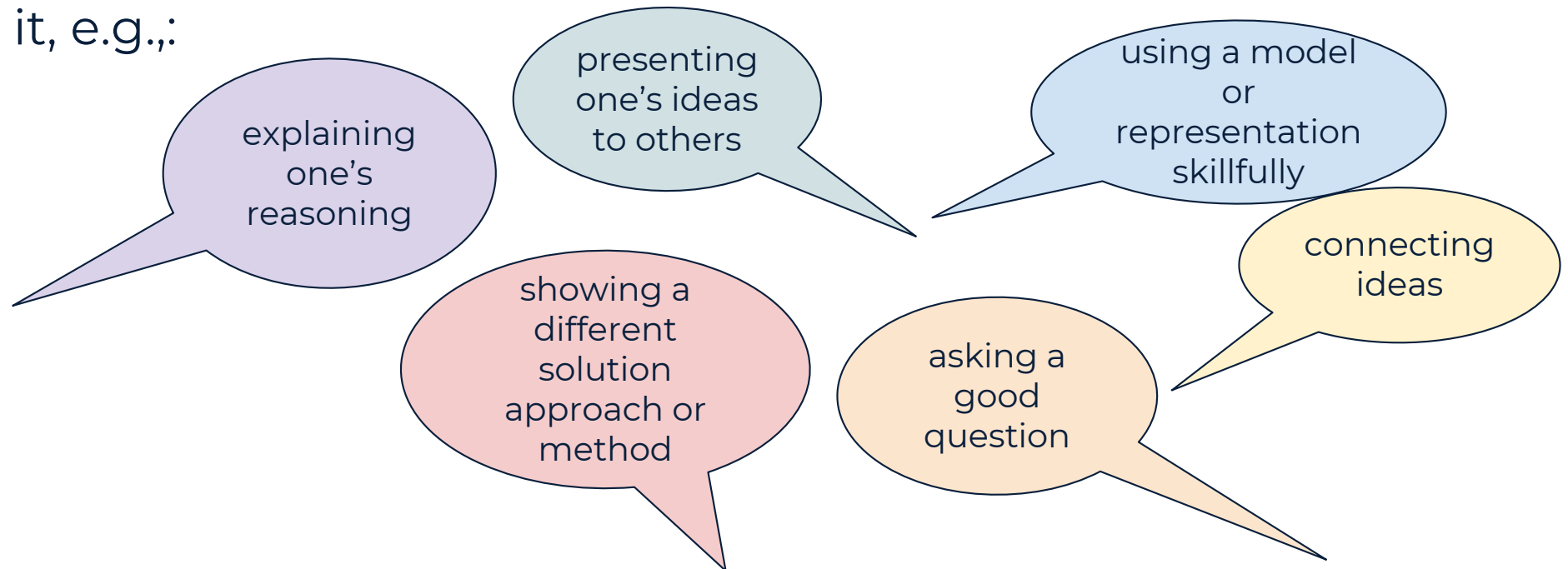
Acknowledging competence

- ★ “It was particularly clear how you used your drawing to explain your thinking.”
- ★ “Belin gave a clear and specific mathematical explanation.”
- ★ “You solved that in a really interesting way. Can you tell us more about your thinking?”
- ★ “Ibn used a very interesting method to show that there are no more solutions. Who can say what Ibn did?”
- ★ “It is clear how closely you are following other people’s thinking and connecting it to the idea you had.”
- ★ “One thing that was really important about what Laken did was to use the definition we developed.”



Practicing acknowledging competence

Identify **specific examples** of mathematics learning competence and **what you might say** to acknowledge it, e.g.,:



Acknowledging competence is a disruptive practice

Disrupting patterns of:

- Narrow, reductive, and distorted views of mathematics
- “Under-teaching”
- Who is seen as mathematically competent
- Negative and deficit views students carry of themselves



What is involved in acknowledging competence?

1. Recognizing and broadening conceptions of what comprises mathematics and mathematical competence, and seeing that in children
2. Understanding the broader systemic factors that shape everyday micro-interactions related to racism and other forms of oppression (sexism, linguicism, ableism, classism) and seeing risks and status hierarchies in one's own class.
3. **Based on #1 and #2, intervening explicitly to acknowledge particular students' competence.**



Two possible ways to acknowledge competence

1. Name a specific competence that a particular student displayed in something they did or said and why that is important in doing math.
2. Ask the class what a particular student did that is a really important thing to do in doing math and what is valuable about that.

Remember to be broadening conceptions of math and mathematical competence.

Other possibilities exist.

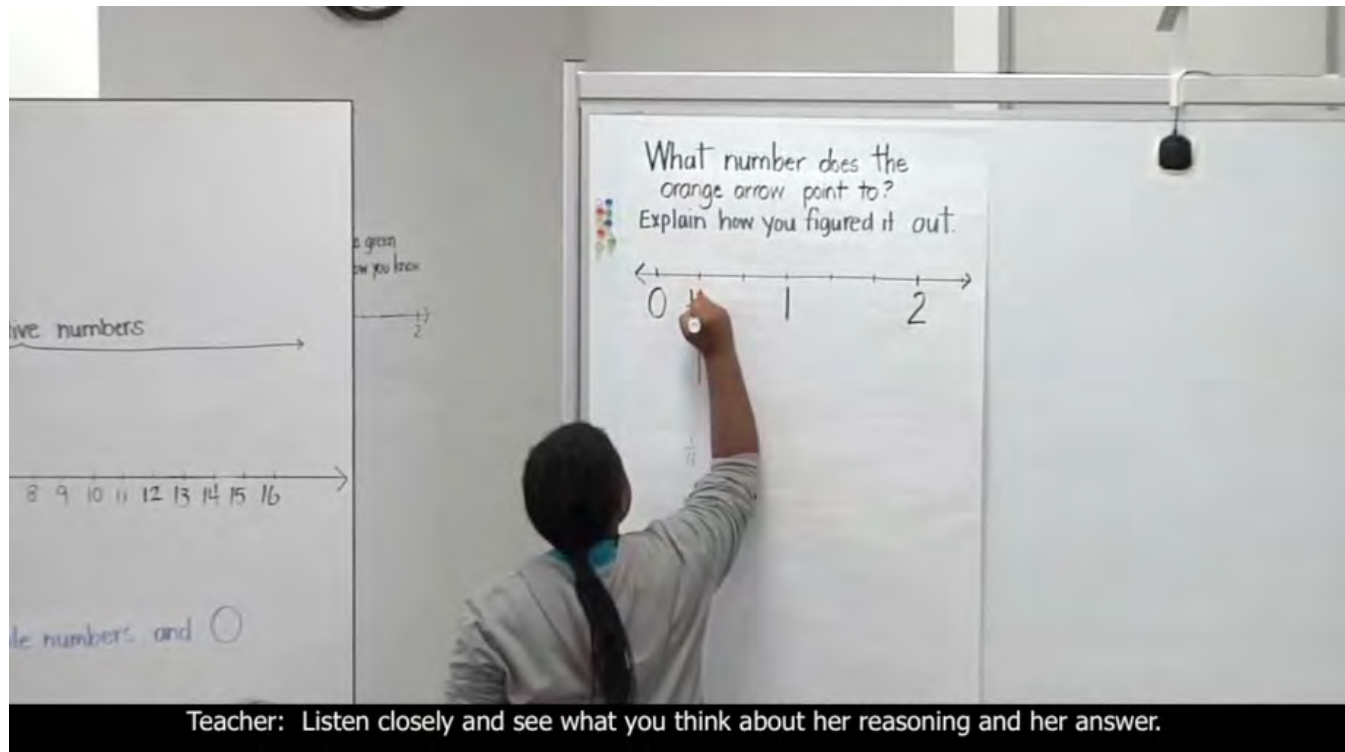


Practice

1. What competences would you acknowledge here?
2. Whom would you choose to acknowledge and why?
3. What would you actually say?



Video: Aniyah and Toni



This video and additional supporting materials are available online [here](#).



Discretionary space #2: Acknowledging competence

Provide opportunities for teachers to:

- Broaden their own understanding of mathematics and what comprises mathematical competence
- Use the concept of “positioning” to intervene to influence who (and what) is seen as competent
- Develop a repertoire for and practice making these moves in authentic and well-timed ways; have ways to do that that are sensitive

3. Engaging students in explanation and reasoning



For students to flourish, they have to do the learning

- Even very good instruction, with high-quality materials, won't lead to student growth
- Students have to be engaged and thinking
- Students are the ones who have to do the work

This is difficult. What are teacher moves that can boost and support students doing the real thinking?



Many teachers . . .

- Use small group instruction
- Have students answering tasks on tablets
- Use exit tickets
- Use turn and talk strategies

But not many successfully get their students doing the thinking collectively.



Mr. Montgomery's class has a two-hour science lab. He gives each student a dish with one cell in it. He tells the class that in twenty minutes, the cell will divide into two cells and each twenty minutes after that, each cell in the dish will divide into two cells.

Time (minutes)	0	20	40	60	80	100	120
Number of cells	1	2	4				
Number of cells as a power of 2	2^0	2^1					



Video available [here](#)

Discretionary space #3: Engaging students in explanation and reasoning

Provide opportunities for teachers to:

- Observe one another
- Study videos to identify and name useful moves; adding to ones that they already use
- Create video clubs to discuss one another's moves

4. Using language to explain to students



Even with a scripted curriculum, teachers have to say and explain things.

Significant mathematical slips are likely, which inadvertently “misteach.”

Teachers might not have ways to accurately explain things that they only memorized.

What is 7
divided by 0?

What is wrong with each of these common statements?

You can't take the square root of a negative number.

There are more fractions than integers.

In a triangle,
 $a^2 + b^2 = c^2$

0 is neither even or odd.

Discretionary space #4: Using language to explain to students

- Have opportunities to practice around common habits of language that create misunderstanding.
- Develop more awareness of this risk.
- Practice developing helpful ways to explain things that do not distort.

Where are the discretionary spaces in **YOUR work** that matter for students' mathematical flourishing?



- Ensuring pacing and coverage/going in depth
- Test taking skills/developing understanding and fluency
- Following the scripted curriculum/adapting to students
- Emphasizing test scores/developing positive mathematical identities

Dilemmas and tensions and your discretionary spaces

BCPS goals

- To improve district mathematical performance
- To support students to develop mathematical competence and positive mathematical identities



How do you use your discretionary spaces in your leadership practice?

- How do you lead in ways to manage these dilemmas and tensions?
- What signals do you send to teachers about each of these?

- Ensuring pacing and coverage/going in depth
- Test taking skills/developing understanding and fluency
- Following the scripted curriculum/adapting to students
- Emphasizing test scores/developing positive mathematical identities



THANK YOU!



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Credits



Image on slide 2:

Photo from “Former Uber security chief to face wire fraud charges for 2016 hack” by Vish Gain, *SiliconRepublic.com*.

Retrieved from <https://www.siliconrepublic.com/enterprise/uber-hack-data-breach-joe-sullivan-security-chief-wire-fraud>



Image on slide 2:

Photo from “What Is a Functional Behavior Assessment & Is It Effective?” by Rachel Wise, *Education and Behavior*.

Retrieved from <https://educationandbehavior.com/steps-for-conducting-a-functional-behavior-assessment/>



Image on slide 2:

Screenshot of Baltimore City Public Schools homepage.

Retrieved from <https://www.baltimorecityschools.org/>

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Image on slides 16–18:
Photo from “Why You Need an Experienced Real Estate Agent” by Elizabeth Weintraub, the balance.
Retrieved from <https://www.thebalance.com/experienced-real-estate-agents-1798883>



Image on slides 16–18:
Photo from “Black doctors earn less than white peers, study reveals” by Stacy M. Brown, The Philadelphia Tribune.
Retrieved from https://www.phillytrib.com/lifestyle/health/black-doctors-earn-less-than-white-peers-study-reveals/article_2d359910-85c8-524b-b6d2-15d1705ad762.html



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Retrieved from <https://whyy.org/articles/want-to-be-a-poll-worker-philly-says-its-responding-to-unprecedented-number-of-applicants/>



Image on slides 16–18:

Photo from “Trump Rioters Storm U.S. Capitol (photos),” Variety

Retrieved from <https://variety.com/gallery/trump-rioters-storm-us-capitol-photos/congress-electoral-college-8/>



Image on slides 17–18:

Photo from “20 Classroom Interventions for Children with Anxiety Disorders” by Jerry Kennard, Ph.D., HealthCentral.

Retrieved from <https://www.healthcentral.com/article/20-classroom-interventions-for-children-with-anxiety-disorders>

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Image on slide 18:

Photo from "10 Ways Well-Meaning White Teachers Bring Racism Into Our Schools" by Jamie Utt, *everyday feminism*.

Retrieved from <https://everydayfeminism.com/2015/08/10-ways-well-meaning-white-teachers-bring-racism-into-our-schools/>

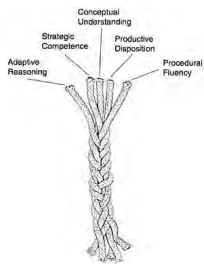


Image on slide 56:

"The strands of mathematical proficiency." National Research Council. 2001. *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: The National Academies Press. doi: 10.17226/9822.



Image on slide 66:

Photo of Bronwyn Davies.

Retrieved from <http://bronwyndavies.com.au/about-bronwyn>

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